JEE-MAIN EXAM APRIL, 2025

Date: - 02-04-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1 Let A be the set of all functions $f: Z \to Z$ and R be a relation on A such that $R = \{(f, g): f(0) = g(1) \text{ and } f(1) = g(0)\}$. Then R is :

(1) Transitive but neither reflexive nor symmetric

(2) Symmetric and transitive but not reflective

(3) Symmetric but neither reflective nor transitive

(4) Reflexive but neither symmetric nor transitive

2. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2 z}{k+\bar{z}} = kz, k \in R$, then the maximum distance of $k + ik^2$ from the circle $|z-(1+2i)| = 1$ is :

(1) $\sqrt{5}+1$ (2) $\sqrt{3}+1$ (3) 3 (4) 2

3. Let $f: R \to R$ be a twice differentiable function such that

$(\sin x \cos y)(f(2x+2y) - f(2x-2y)) = (\cos x \sin y)(f(2x+2y) + f(2x-2y))$, for all $x, y \in R$.

If $f'(0) = \frac{1}{2}$, then the value of $24 f''\left(\frac{5\pi}{3}\right)$ is :

(1) -2 (2) 2 (3) 3 (4) -3

4. Let $P_n = \alpha^n + \beta^n, n \in N$. If $P_{10} = 123, P_9 = 76, P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(1) $x^2 - x + 1 = 0$ (2) $x^2 - x - 1 = 0$ (3) $x^2 + x + 1 = 0$ (4) $x^2 + x - 1 = 0$

5. Let $a_1, a_2, a_3, \ldots$ be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5} a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is :

(1) 18 (2) 10 (3) 17 (4) 11

6. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x axis, where P lies in the first quadrant. If the circle, whose one diameter is PS , S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

(1) 20 (2) 30 (3) 25 (4) 15

7. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}, QR=5$ and the coordinates of the point P be $(0,2,3)$. If the area of the triangle PQR is $\frac{m}{n}$ then:

(1) $2m-5\sqrt{21}n=0$ (2) $5m-2\sqrt{21}n=0$ (3) $5m-21\sqrt{2}n=0$ (4) $m-5\sqrt{21}n=0$

8. The term independent of x in the expansion of $\left(\frac{(x+1)}{\left(x^{2/3}+1-x^{1/3}\right)}-\frac{(x-1)}{\left(x-x^{1/2}\right)}\right)^{10}, x>1$, is :

(1) 120 (2) 240 (3) 210 (4) 150

9. If $\vec{a}$ is a nonzero vector such that its projections on the vectors $2\hat{i}-\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is :

(1) $\frac{1}{\sqrt{155}}(-7\hat{i}+9\hat{j}+5\hat{k})$ (2) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}+5\hat{k})$

(3) $\frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}-5\hat{k})$ (4) $\frac{1}{\sqrt{155}}(-7\hat{i}+9\hat{j}-5\hat{k})$

10. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta+(2-\sqrt{6})\cos\theta-\sqrt{3}=0$, is equal to :

(1) 8 (2) 12 (3) 10 (4) 6

11. If the function $f(x)=2x^3-9ax^2+12a^2x+1$, where $a>0$, attains its local maximum and local minimum values at p and q , respectively, such that $p^2=q$, then $f(3)$ is equal to :

(1) 10 (2) 55 (3) 37 (4) 23

12. If S and S' are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then $\min\left(SP\cdot S'P\right)+\max\left(SP\cdot S'P\right)$ is equal to :

(1) $3(6+\sqrt{2})$ (2) 27 (3) $3(1+\sqrt{2})$ (4) 9

13. The number of sequences of ten terms, whose terms are either 0 or 1 or 2 , that contain exactly five 1 s and exactly three 2 s , is equal to :

(1) 360 (2) 2520 (3) 1820 (4) 45

14. Let $A=\begin{bmatrix}\alpha & -1\\ 6 & \beta\end{bmatrix}, \alpha>0$, such that $\det(A)=0$ and $\alpha+\beta=1$. If I denotes 2×2 identity matrix, then the matrix $(I+A)^8$ is :

(1) $\begin{bmatrix}766 & -255\\ 1530 & -509\end{bmatrix}$ (2) $\begin{bmatrix}257 & -64\\ 514 & -127\end{bmatrix}$ (3) $\begin{bmatrix}4 & -1\\ 6 & -1\end{bmatrix}$ (4) $\begin{bmatrix}1025 & -511\\ 2024 & -1024\end{bmatrix}$

15. The largest $n \in N$ such that 3^n divides 50 ! is :

(1) 23 (2) 21 (3) 22 (4) 20

16. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5,6 and 7 square units respectively. Then the area (in square units) of the ΔBCD is equal to :

(1) $\sqrt{110}$ (2) $\sqrt{340}$ (3) 12 (4) $7\sqrt{3}$

17. For $\alpha, \beta, \gamma \in R$, if $\lim_{x\to 0} \dfrac{x^2 \sin \alpha x + (\gamma - 1)e^{x^2}}{\sin 2x - \beta x} = 3$, then $\beta + \gamma - \alpha$ is equal to :

(1) 6 (2) -1 (3) 4 (4) 7

18. Let $a \in R$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3. If det $((a+1)\text{adj}((a-1)A))$ is $2^m 3^n, m,\ n \in \{0,1,2,\ldots,20\}$, then $m+n$ is equal to :

(1) 14 (2) 16 (3) 17 (4) 15

19. Let one focus of the hyperbola $H : \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ be at $(\sqrt{10}, 0)$ and the corresponding directrix be $x = \dfrac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H, then $9\left(e^2 + l\right)$ is equal to :

(1) 16 (2) 12 (3) 15 (4) 14

20. If the system of linear equations

$3x + y + \beta z = 3$

$2x + \alpha y - z = -3$

$x + 2y + z = 4$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is :

(1) 31 (2) 43 (3) 49 (4) 37

SECTION-B

21. If the area of the region $\left\{(x,y) : \left|4 - x^2\right| \le y \le x^2, y \le 4, x \ge 0\right\}$ is $\left(\dfrac{80\sqrt{2}}{\alpha} - \beta\right), \alpha, \beta \in N$, then $\alpha + \beta$ is equal to _____.

22. The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to _____

23. Let $f: R \to R$ be a thrice differentiable odd function satisfying $f'(x) \geq 0, f''(x) = f(x), f(0) = 0, f'(0) = 3$. Then $9f(\log_e 3)$ is equal to _______.

24. Let [.] denote the greatest integer function. If $\int_0^{e^3} \left[\frac{1}{e^{x-1}}\right] dx = \alpha - \log_e 2$, then α^3 is equal to _____.

25. Three distinct numbers are selected randomly from the set $\{1, 2, 3, \ldots, 40\}$. If the probability, that the selected numbers are in an increasing G.P., is $\frac{m}{n}, \gcd(m, n) = 1$, then $m + n$ is equal to _____ .

NTA ANSWERS

1.	**(3)**	**2.**	**(1)**	**3.**	**(4)**	**4.**	**(4)**	**5.**	**(4)**	**6.**	**(4)**	**7.**	**(1)**
8.	**(3)**	**9.**	**(2)**	**10.**	**(1)**	**11.**	**(3)**	**12.**	**(2)**	**13.**	**(2)**	**14.**	**(1)**
15.	**(3)**	**16.**	**(1)**	**17.**	**(4)**	**18.**	**(2)**	**19.**	**(1)**	**20.**	**(1)**	**21.**	**22**
22.	**768**	**23.**	**36**	**24.**	**8**	**25.**	**4949**						