JEE-MAIN EXAM APRIL, 2025

Date: - 04-04-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1. $1+3+5^2+7+9^2+\ldots$ upto 40 terms is equal to

(1) 40870 (2) 43890 (3) 41880 (4) 33980

2. Let $f:[0,\infty)\to\mathbb{R}$ be a differentiable function such that

$$f(x)=1-2x+\int_0^x e^{x-t}f(t)dt \text{ for all } x\in[0,\infty)$$

Then the area of the region bounded by $y=f(x)$ and the coordinate axes is

(1) $\sqrt{2}$ (2) $\frac{1}{2}$

(3) $\sqrt{5}$ (4) 2

3. The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is

(1) $\frac{103}{182}$ (2) $\frac{17}{26}$

(3) $\frac{19}{26}$ (4) $\frac{129}{182}$

4. Consider two vectors

$\vec{u}=3\hat{i}-\hat{j}$ and $\vec{v}=2\hat{i}+\hat{j}-\lambda\hat{k},\lambda>0$.

The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$.

Let $\vec{v}=\vec{v}_1+\vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v_2}$ is perpendicular to $\vec{u}$. Then the value $\left|\vec{v_1}\right|^2+\left|\vec{v_2}\right|^2$ is equal to

(1) 14 (2) 10

(3) $\frac{25}{2}$ (4) $\frac{23}{2}$

5. If $10\sin^4\theta+15\cos^4\theta=6$, then the value of $\frac{27\,\text{cosec}^6\theta+8\sec^6\theta}{16\sec^8\theta}$ is

(1) $\frac{1}{5}$ (2) $\frac{3}{5}$ (3) $\frac{2}{5}$ (4) $\frac{3}{4}$

6. Let $f, g:(1,\infty)\to\mathbb{R}$ be defined as $f(x)=\frac{2x+3}{5x+2}$ and $g(x)=\frac{2-3x}{1-x}$. If the range of the function fog : $[2,4]\to\mathbb{R}$ is $[\alpha,\beta]$, then $\frac{1}{\beta-\alpha}$ is equal to

(1) 68 (2) 56 (3) 29 (4) 2

7. Let A and B be two distinct points on the line $L:\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1,2,3)$ on the line L. If O is the origin, then $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is equal to

(1) 62 (2) 47 (3) 21 (4) 49

8. Let the shortest distance between the lines $\frac{x-3}{3}=\frac{y-\alpha}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha+\beta$ is

(1) 48 (2) 42 (3) 40 (4) 46

9. For an integer $n\geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x+y)^{2n-3}$ is 16, then the distance of the point $P\left(2n-1, n^2-4n\right)$ from the line $x+y=8$ is

(1) $2\sqrt{2}$ (2) $\sqrt{2}$ (3) $5\sqrt{2}$ (4) $3\sqrt{2}$

10. Let $A=\{1, 6, 11, 16,\ldots\}$ and $B=\{9,16,23,30,\ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A\cup B)$ is

(1) 4027 (2) 3814 (3) 3761 (4) 4003

11. Consider the equation $x^2+4x-n=0$, where $n\in[20,100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

(1) 7 (2) 6 (3) 5 (4) 8

12. Consider the sets $A=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x^2+y^2=25\right\}, B=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x^2+9y^2=144\right\}$, $C=\left\{(x,y)\in\mathbb{Z}\times\mathbb{Z}: x^2+y^2\leq 4\right\}$ and $D=A\cap B$. The total number of one-one functions from the set D to the set C is:

(1) 17160 (2) 15120 (3) 18290 (4) 19320

13. Considering the principal values of the inverse trigonometric functions,

$\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2}<x<\frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{5\pi}{6}-\sin^{-1}x$ (2) $\frac{\pi}{4}+\sin^{-1}x$

(3) $\frac{-5\pi}{6}-\sin^{-1}x$ (4) $\frac{\pi}{6}+\sin^{-1}x$

14. If $\lim\limits_{x\to 1^+}\dfrac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3}=-1$, where $\lambda,\mu\in\mathbb{R}$, then $\lambda+\mu$ is equal to

(1) 18 (2) 19 (3) 17 (4) 20

15. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function satisfying $f(0)=1$ and $f(2x)-f(x)=x$ for all $x\in\mathbb{R}$. If $\lim\limits_{n\to\infty}\left\{f(x)-f\left(\dfrac{x}{2^n}\right)\right\}=G(x)$, then $\sum\limits_{r=1}^{10}G\left(r^2\right)$ is equal to

(1) 215 (2) 540 (3) 420 (4) 385

16. The value of $\int_{-1}^{1}\dfrac{(1+\sqrt{|x|-x})e^x+(\sqrt{|x|-x})e^{-x}}{e^x+e^{-x}}dx$ is equal to

(1) $1+\dfrac{2\sqrt{2}}{3}$ (2) $2+\dfrac{2\sqrt{2}}{3}$

(3) $3-\dfrac{2\sqrt{2}}{3}$ (4) $1-\dfrac{2\sqrt{2}}{3}$

17. Let the three sides of a triangle are on the lines $4x-7y+10=0, x+y=5$ and $7x+4y=15$. Then the distance of its orthocentre from the orthocentre of the tringle formed by the lines $x=0, y=0$ and $x+y=1$ is

(1) $\sqrt{5}$ (2) 5 (3) $\sqrt{20}$ (4) 20

18. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is

(1) $\dfrac{2}{15}$ (2) $\dfrac{28}{75}$ (3) $\dfrac{11}{15}$ (4) $\dfrac{3}{5}$

19. In the expansion of $\left(\sqrt[3]{2}+\dfrac{1}{\sqrt[3]{3}}\right)^n, n\in\mathrm{N}$, if the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\dfrac{1}{6}$, then the value of ${}^nC_3$ is

(1) 2300 (2) 1040 (3) 4060 (4) 4960

20. The length of the latus-rectum of the ellipse, whose foci are $(2,5)$ and $(2,-3)$ and eccentricity is $\dfrac{4}{5}$, is

(1) $\dfrac{6}{5}$ (2) $\dfrac{10}{3}$ (3) $\dfrac{18}{5}$ (4) $\dfrac{50}{3}$

SECTION-B

21. Let m and n be the number of points at which the function $f(x)=\max\left\{x,x^3,x^5,\ldots,x^{21}\right\}, x\in\mathbb{R}$, is not differentiable and not continuous, respectively. Then $m+n$ is equal to _____

22. If the area of the region $\{(x,y):|x-5|\le y\le 4\sqrt{x}\}$ is A , then 3A is equal to _____

23. Let $A=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$. If for some $\theta\in(0,\pi), A^2=A^T$, then the sum of the diagonal elements of the matrix $(A+I)^3+(A-I)^3-6A$ is equal to _____.

24. Let $A=\{z\in C:|z-2-i|=3\}, B=\{z\in C:\mathrm{Re}(z-iz)=2\}$ and $S=A\cap B$. Then $\sum_{z\in S}|z|^2$ is equal to _____

25. Let C be the circle $x^2+(y-1)^2=2, E_1$ and E_2 be two ellipses whose centres lie at the origin and 2 major axes lie on x-axis and y-axis respectively. Let the straight line $x+y=3$ touch the curves C, E_1 and E_2 at $P(x_1,y_1), Q(x_2,y_2)$ and $R(x_3,y_3)$ respectively. Given that P is the mid point of the line segment QR and $PQ=\frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1+x_2y_2+x_3y_3)$ is equal to _____

NTA ANSWERS

1.	**(3)**	**2.**	**(2)**	**3.**	**(4)**	**4.**	**(1)**	**5.**	**(3)**	**6.**	**(2)**	**7.**	**(2)**
8.	**(4)**	**9.**	**(4)**	**10.**	**(3)**	**11.**	**(2)**	**12.**	**(1)**	**13.**	**(4)**	**14.**	**(1)**
15.	**(4)**	**16.**	**(1)**	**17.**	**(1)**	**18.**	**(2)**	**19.**	**(1)**	**20.**	**(3)**	**21.**	**(3)**
22.	**(368)**	**23.**	**(6)**	**24.**	**(22)**	**25.**	**(46)**						