JEE-MAIN EXAM APRIL, 2025

Date: - 04-04-2025 (SHIFT-2)

MATHEMATICS

SECTION-A

1. The axis of a parabola is the line $y=x$ and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point $(1,k)$ lies on the parabola, then a possible value of k is :

(1) 8 (2) 3 (3) 4 (4) 9

2. Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$ and $\vec{r}=(p\hat{i}+2\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be $a,b,(a<b)$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :

(1) $\frac{2}{3}$ (2) 18 (3) 9 (4) $\frac{3}{2}$

3. Let the matrix $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ satisfy $A^n=A^{n-2}+A^2-1$ for $n\geqslant 3$. Then the sum of all the elements of A^{50} is :

(1) 39 (2) 44 (3) 53 (4) 52

4. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of $\mathrm{AP}'\mathrm{s}$ in A and B respectively such that $D=d+3, d>0$. If $\frac{p+q}{p-q}=\frac{19}{5}$, then $p-q$ is equal to

(1) 630 (2) 540 (3) 450 (4) 600

5. If $1^2\cdot\left({}^{15}C_1\right)+2^2\cdot\left({}^{15}C_2\right)+3^2\cdot\left({}^{15}C_3\right)+\ldots+15^2\cdot\left({}^{15}C_{15}\right)=2^m\cdot 3^n\cdot 5^k$, where $m,n,k\in\mathrm{N}$, then $m+n+k$ is equal to :

(1) 20 (2) 21 (3) 18 (4) 19

6. Let f be a differentiable function on $\mathbf{R}$ such that $f(2)=1, f'(2)=4$. Let $\lim_{x\to 0}(f(2+x))^{3/x}=\mathbf{e}^{\alpha}$.

Then the number of times the curve $y=4x^3-4x^2-4(\alpha-7)x-\alpha$ meets x-axis is :

(1) 1 (2) 3 (3) 0 (4) 2

7. Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a, b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

(1) $\frac{3}{4}$ (2) 2

(3) $\frac{1}{2}$ (4) 1

8. Let A be the point of intersection of the lines $L_1:\frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}$ and $L_2:\frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$. Let B and C be the points on the lines L_1 and L_2 respectively such that $AB=AC=\sqrt{15}$. Then the square of the area of the triangle ABC is :

(1) 57 (2) 60 (3) 63 (4) 54

9. Let for two distinct values of p the lines $y=x+p$ touch the ellipse $E:\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ at the points A and B. Let the line $y=x$ intersect E at the points C and D. Then the area of the quadrilateral ABCD is equal to :

(1) 24 (2) 20 (3) 48 (4) 36

10. If the sum of the first 20 terms of the series

$$\frac{4\cdot 1}{4+3\cdot 1^2+1^4}+\frac{4\cdot 2}{4+3\cdot 2^2+2^4}+\frac{4\cdot 3}{4+3\cdot 3^2+3^4}+\frac{4\cdot 4}{4+3\cdot 4^2+4^4}+\ldots \text{ is } \frac{m}{n},$$

where m and n are coprime, then $m+n$ is equal to :

(1) 423 (2) 420 (3) 422 (4) 421

11. A line passing through the point $A(-2,0)$, touches the parabola $P:y^2=x-2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is :

(1) $\frac{7}{3}$ (2) $\frac{8}{3}$ (3) 3 (4) 2

12. If a curve $y=y(x)$ passes through the point $\left(1,\frac{\pi}{2}\right)$ and satisfies the differential equation $\left(7x^4\cot y-e^x\operatorname{cosec} y\right)\frac{dx}{dy}=x^5, x\ge 1$, then at $x=2$, the value of $\cos y$ is :

(1) $\frac{2e^2-e}{128}$ (2) $\frac{2e^2-e}{64}$ (3) $\frac{2e^2+e}{128}$ (4) $\frac{2e^2+e}{64}$

13. Let $A=\{-3,-2,-1,0,1,2,3\}$ and R be a relation on A defined by $x\text{R}y$ if and only if $2x-y\in\{0,1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l+\text{m}+\text{n}$ is equal to :

(1) 15 (2) 16 (3) 18 (4) 17

14. Let $f(x)+2f\left(\frac{1}{x}\right)=x^2+5$ and $2g(x)-3g\left(\frac{1}{2}\right)=x, x>0$. If $\alpha=\int_1^2 f(x)dx$, and $\beta=\int_1^2 g(x)dx$, then the value of $9\alpha+\beta$ is :

(1) 11 (2) 10 (3) 0 (4) 1

15. The centre of a circle C is at the centre of the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle PF_1F_2 is 30 and the length of the major axis of E is 17, then the distance between the foci of E is :

(1) 26 (2) 12 (3) $\frac{13}{2}$ (4) 13

16. Let the product of $\omega_1=(8+i)\sin\theta+(7+4i)\cos\theta$ and $\omega_2=(1+8i)\sin\theta+(4+7i)\cos\theta$ be $\alpha+i\beta$, $i=\sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha+\beta$ respectively. Then $p+q$ is equal to :

(1) 150 (2) 160 (3) 140 (4) 130

17. Let the domains of the functions $f(x)=\log_4\log_3\log_7\left(8-\log_2\left(x^2+4x+5\right)\right)$ and $\text{g}(x)=\sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α,β) and $[\gamma,\delta]$, respectively. Then $\alpha^2+\beta^2+\gamma^2+\delta^2$ is equal to :

(1) 13 (2) 14 (3) 16 (4) 15

18. Let the sum of the focal distances of the point $\text{P}(4,3)$ on the hyperbola $\text{H}:\frac{x^2}{\text{a}^2}-\frac{y^2}{\text{b}^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distances of the point P is m , then $9l^2+6\text{m}$ is equal to :

(1) 184 (2) 185 (3) 186 (4) 187

19. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)+\ldots$ is :

(1) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$ (2) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$

(3) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$ (4) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$

20. Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1x_2 = 54$, then $a + x_1 + x_2$ is equal to :

(1) 15 (2) 13 (3) 18 (4) 24

SECTION-B

21. Let the three sides of a triangle ABC be given by the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2\right)$ is equal to ___________.

22. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{k=1}^{n}\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20$, then n is equal to ___________.

23. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, then n is equal to ___________.

24. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}dx = \frac{1}{m}\left(\left(\sqrt{1+x^2}+x\right)^n\left(\pi\sqrt{1+x^2}-x\right)\right) + C$ where C is the constant of integration and $m, n \in \mathbf{N}$, then $m + n$ is equal to ___________.

25. Let m and n, $(m < n)$, be two 2-digit numbers. Then the total numbers of pairs (m, n), such that $\gcd(m, n) = 6$, is ___________.

NTA ANSWERS

1.	**(4)**	**2.**	**(1)**	**3.**	**(3)**	**4.**	**(2)**	**5.**	**(4)**	**6.**	**(4)**	**7.**	**(4)**
8.	**(4)**	**9.**	**(1)**	**10.**	**(4)**	**11.**	**(2)**	**12.**	**(1)**	**13.**	**(4)**	**14.**	**(1)**
15.	**(4)**	**16.**	**(4)**	**17.**	**(4)**	**18.**	**(2)**	**19.**	**(1)**	**20.**	**(3)**	**21.**	**(164)**
22.	**(11)**	**23.**	**(2)**	**24.**	**(379)**	**25.**	**(64)**						