JEE-MAIN EXAM APRIL, 2024

Date: - 04-04-2024 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Let $f: \mathrm{R} \rightarrow \mathrm{R}$ be a function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2} & , x<0 \\ \alpha & , x=0 \\ \dfrac{\beta\sqrt{1-\cos x}}{x} & , x>0\end{cases}$$, where $\alpha, \beta \in R$. If

f is continuous at $\mathrm{x}=0$, then $\alpha^2+\beta^2$ is equal to :

(1) 48 (2) 12 (3) 3 (4) 6

2. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\frac{4}{17}$ (2) $\frac{5}{18}$ (3) $\frac{7}{18}$ (4) $\frac{5}{16}$

3. The vertices of a triangle are $\mathrm{A}(-1,3), \mathrm{B}(-2,2)$ and $\mathrm{C}(3,-1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

(1) $x-y-(2+\sqrt{2})=0$ (2) $-x+y-(2-\sqrt{2})=0$

(3) $x+y-(2-\sqrt{2})=0$ (4) $x+y+(2-\sqrt{2})=0$

4. If the solution $y=y(x)$ of the differential equation $(x^4+2x^3+3x^2+2x+2)dy-(2x^2+2x+3)dx=0$ satisfies $y(-1)=-\frac{\pi}{4}$, then $y(0)$ is equal to :

(1) $-\frac{\pi}{12}$ (2) 0 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

5. Let the sum of the maximum and the minimum values of the function $f(x)=\frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where $\gcd(\mathrm{m}, \mathrm{n})=1$. Then $\mathrm{m}+\mathrm{n}$ is equal to :

(1) 182 (2) 217 (3) 195 (4) 201

6. One of the points of intersection of the curves $y=1+3x-2x^2$ and $y=\frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}(\ell\sqrt{5}+\mathrm{m})-\mathrm{n}\log_e(1+\sqrt{5})$, where $\ell, \mathrm{m}, \mathrm{n} \in \mathrm{N}$. Then $\ell+\mathrm{m}+\mathrm{n}$ is equal to

(1) 32 (2) 30 (3) 29 (4) 31

7. If the system of equations

$x+(\sqrt{2}\sin\alpha)y+(\sqrt{2}\cos\alpha)z=0$

$\mathrm{x}+(\cos\alpha)\mathrm{y}+(\sin\alpha)\mathrm{z}=0$

$\mathrm{x}+(\sin\alpha)\mathrm{y}-(\cos\alpha)\mathrm{z}=0$

has a non-trivial solution, then $\alpha\in\left(0,\frac{\pi}{2}\right)$ is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$ (3) $\frac{5\pi}{24}$ (4) $\frac{11\pi}{24}$

8. There are 5 points $\mathrm{P}_1,\mathrm{P}_2,\mathrm{P}_3,\mathrm{P}_4,\mathrm{P}_5$ on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points $\mathrm{P}_6,\mathrm{P}_7,\ldots,\mathrm{P}_{11}$ on the side BC and 7 points $\mathrm{P}_{12},\mathrm{P}_{13},\ldots,\mathrm{P}_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $\mathrm{P}_1,\mathrm{P}_2,\ldots,\mathrm{P}_{18}$ as vertices, is :

(1) 776 (2) 751 (3) 796 (4) 771

9. Let $f(x)=\begin{cases}-2, & -2\le x\le 0\\ x-2, & 0<x\le 2\end{cases}$ and $h(x)=f(|x|)+|f(x)|$. Then $\int_{-2}^{2}\mathrm{h(x)dx}$ is equal to :

(1) 2 (2) 4 (3) 1 (4) 6

10. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 3133 (2) 633 (3) 931 (4) 6131

11. Let a unit vector which makes an angle of 60° with $2\hat{\imath}+2\hat{\jmath}-\hat{k}$ and an angle of 45° with $\hat{\imath}-\hat{k}$ be $\vec{C}$. Then $\vec{\mathrm{C}}+\left(-\frac{1}{2}\hat{\mathrm{i}}+\frac{1}{3\sqrt{2}}\hat{\mathrm{j}}-\frac{\sqrt{2}}{3}\hat{\mathrm{k}}\right)$ is :

(1) $-\frac{\sqrt{2}}{3}\hat{\imath}+\frac{\sqrt{2}}{3}\hat{\jmath}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$ (2) $\frac{\sqrt{2}}{3}\hat{\mathrm{i}}+\frac{1}{3\sqrt{2}}\hat{\mathrm{j}}-\frac{1}{2}\hat{\mathrm{k}}$

(3) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{\mathrm{i}}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{\mathrm{j}}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{\mathrm{k}}$ (4) $\frac{\sqrt{2}}{3}\hat{\mathrm{i}}-\frac{1}{2}\hat{\mathrm{k}}$

12. Let the first three terms $2,p$ and q, with $q\neq 2$, of a G.P. be respectively the 7^{th} , 8^{th} and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to

(1) 151 (2) 169 (3) 177 (4) 163

13. Let $a,b\in R$. Let the mean and the variance of 6 observations $-3,4,7,-6,\mathrm{a},\mathrm{b}$ be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{13}{3}$ (2) $\frac{16}{3}$ (3) $\frac{11}{3}$ (4) $\frac{14}{3}$

14. If 2 and 6 are the roots of the equation $\mathrm{ax}^2+\mathrm{bx}+1=0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(1) $2x^2+11x+12=0$ (2) $4x^2+14x+12=0$

(3) $x^2+10x+16=0$ (4) $x^2+8x+12=0$

15. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2+|z|=0, z\in C$. Then $4(\alpha^2+\beta^2)$ is equal to :

(1) 6 (2) 4 (3) 8 (4) 2

16. Let the point, on the line passing through the points $\mathrm{P}(1,-2,3)$ and $\mathrm{Q}(5,-4,7)$, farther from the origin and at a distance of 9 units from the point P, be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 155 (2) 150 (3) 160 (4) 165

17. A square is inscribed in the circle $x^2+y^2-10x-6y+30=0$. One side of this square is parallel to $y=x+3$. If (x_i,y_i) are the vertices of the square, then $\sum(x_i^2+y_i^2)$ is equal to :

(1) 148 (2) 156 (3) 160 (4) 152

18. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-1}\right)$ is $(\alpha,\beta]$, then $3\alpha+10\beta$ is equal to :

(1) 97 (2) 100 (3) 95 (4) 98

19. Let $f(x)=x^5+2e^{x/4}$ for all $x\in R$. Consider a function $g(x)$ such that (gof) $(x)=x$ for all $x\in R$. Then the value of $8\,g'(2)$ is :

(1) 16 (2) 4 (3) 8 (4) 2

20. Let $\alpha\in(0,\infty)$ and $A=\begin{bmatrix}1 & 2 & \alpha\\ 1 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}$.

If $\det\left(\operatorname{adj}(2A-A^T)\cdot\operatorname{adj}(A-2A^T)\right)=2^8$, then $(\det(A))^2$ is equal to :

(1) 1 (2) 49 (3) 16 (4) 36

SECTION-B

21. If $\lim_{x\to1}\frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}=\frac{m\sqrt{5}}{n(2n)^{2/3}}$, where $\gcd(m,n)=1$, then $8m+12n$ is equal to

22. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then $m+n$ is equal to

23. Let the solution $y=y(x)$ of the differential equation $\frac{dy}{dx}-y=1+4\sin x$ satisfy $y(\pi)=1$. Then $y\left(\frac{\pi}{2}\right)+10$ is equal to

24. If the shortest distance between the lines $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-5}{4}$ and $\frac{x-3}{1}=\frac{y-2}{-3}=\frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$ and $\int_0^k[x^2]dx=\alpha-\sqrt{\alpha}$, where $[x]$ denotes the greatest integer function, then $6\alpha^3$ is equal to

25. Let A be a square matrix of order 2 such that $|A|=2$ and the sum of its diagonal elements is -3. If the points (x,y) satisfying $A^2+xA+yI=0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is ℓ, then $e^4+\ell^4$ is equal to

26. Let $a=1+\frac{{}^2C_2}{3!}+\frac{{}^3C_2}{4!}+\frac{{}^4C_2}{5!}+\cdots$,

$$b=1+\frac{{}^1C_0+{}^1C_1}{1!}+\frac{{}^2C_0+{}^2C_1+{}^2C_2}{2!}+\frac{{}^3C_0+{}^3C_1+{}^3C_2+{}^3C_3}{3!}+\cdots$$

Then $\frac{2b}{a^2}$ is equal to

27. Let A be a 3×3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix}=3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of $\det(A)$ is

28. Let the length of the focal chord PQ of the parabola $\mathrm{y}^2 = 12\mathrm{x}$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to

29. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{\mathrm{AB}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} - 7\hat{\mathrm{k}}$, $\overrightarrow{\mathrm{BC}} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\overrightarrow{\mathrm{AC}} = 6\hat{\mathrm{i}} + \mathrm{d}\hat{\mathrm{j}} - 2\hat{\mathrm{k}}, \mathrm{d} > 0$. Then the square of the length of the largest side of the triangle ABC is

30. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\mathrm{si}\quad x\cos x} dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where a, $\mathrm{b} \in \mathrm{N}$, then $\mathrm{a} + \mathrm{b}$ is equal to

NTA ANSWER

1.	**(2)**	**2.**	**(2)**	**3.**	**(3)**	**4.**	**(3)**	**5.**	**(4)**
6.	**(2)**	**7.**	**(3)**	**8.**	**(2)**	**9.**	**(1)**	**10.**	**(1)**
11.	**(4)**	**12.**	**(4)**	**13.**	**(1)**	**14.**	**(4)**	**15.**	**(2)**
16.	**(1)**	**17.**	**(4)**	**18.**	**(1)**	**19.**	**(1)**	**20.**	**(3)**
21.	**(100)**	**22.**	**(45)**	**23.**	**(7)**	**24.**	**(48)**	**25.**	**(25)**
26.	**(8)**	**27.**	**(27)**	**28.**	**(72)**	**29.**	**(54)**	**30.**	**(8)**