JEE-MAIN EXAM APRIL, 2024

Date: - 05-04-2024 (SHIFT-2)

MATHEMATICS

SECTION-A

1. Let $f:[-1,2] \rightarrow \mathrm{R}$ be given by $f(\mathrm{x}) = 2\mathrm{x}^2 + \mathrm{x} + [\mathrm{x}^2] - [\mathrm{x}]$, where $[\mathrm{t}]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(1) 6 (2) 3 (3) 4 (4) 5

2. The differential equation of the family of circles passing the origin and having center at the line $\mathrm{y} = \mathrm{x}$ is :

(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$ (2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$ (4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

3. Let $S_1 = \{z \in C : |z| \leq 5\}$, $S_2 = \left\{z \in C : \mathrm{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \geq 0\right\}$ and $S_3 = \{z \in C : \mathrm{Re}(z) \geq 0\}$. Then

(1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{24}$ (3) $\frac{125\pi}{4}$ (4) $\frac{125\pi}{12}$

4. The area enclosed between the curves $y = x|x|$ and $\mathrm{y} = \mathrm{x} - |\mathrm{x}|$ is :

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$ (3) 1 (4) $\frac{4}{3}$

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is :

(1) OBBHJ (2) HBBJO (3) OBBJH (4) JBBOH

6. Let $\vec{a} = 2\hat{\imath} + 5\hat{\jmath} - \hat{k}, \vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $\vec{c}$ be three vectors such that $(\vec{c} + \hat{\imath}) \times (\vec{a} + \vec{b} + \hat{\imath}) = \vec{a} \times (\vec{c} + \hat{\imath}) \cdot \vec{a} \cdot \vec{c} = -29$, then $\vec{c} \cdot (-2\hat{\imath} + \hat{\jmath} + \hat{k})$ is equal to :

(1) 10 (2) 5 (3) 15 (4) 12

7. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to :

(1) 110 (2) 105 (3) 124 (4) 121

8. Let $\mathrm{A}(-1,1)$ and $\mathrm{B}(2,3)$ be two points and P be a variable point above the line AB such that the area of $\triangle \mathrm{PAB}$ is 10 . If the locus of P is $\mathrm{ax} + \mathrm{by} = 15$, then $5a + 2b$ is :

(1) $-\frac{12}{5}$ (2) $-\frac{6}{5}$ (3) 4 (4) 6

9. Let (α, β, γ) be the point $(8,5,7)$ in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to

(1) 16 (2) 18 (3) 14 (4) 20

10. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}, x \neq 0$, is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to :

(1) 639 (2) 724 (3) 693 (4) 742

11. Let $f, g: \mathrm{R} \to \mathrm{R}$ be defined as : $f(x) = |x-1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$. Then the function $f(g(x))$ is

(1) neither one-one nor onto. (2) one-one but not onto.

(3) both one-one and onto. (4) onto but not one-one.

12. Let the circle $C_1: x^2 + y^2 - 2(x+y) + 1 = 0$ and C_2 be a circle having centre at $(-1,0)$ and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

(1) 2 (2) 1 (3) 6 (4) 4

13. Let the set $S = \{2,4,8,16, \ldots, 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

(1) 1680 (2) 1520 (3) 1710 (4) 1640

14. The values of m, n, for which the system of equations

$x + y + z = 4,$

$2x + 5y + 5z = 17,$

$x + 2y + mz = n$

has infinitely many solutions, satisfy the equation :

(1) $m^2 + n^2 - m - n = 46$ (2) $m^2 + n^2 + m + n = 64$

(3) $m^2 + n^2 + mn = 68$ (4) $m^2 + n^2 - mn = 39$

15. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set $\{1,2,3,4,5,6\}$. If the probability of this equation having one real root bigger than the other is p, then 216 p equals :

(1) 57 (2) 38 (3) 19 (4) 76

16. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

(1) $r = 1$ (2) $r^2 - 8r + 8 = 0$ (3) $2r^2 - 4r + 1 = 0$ (4) $2r^2 - 8r + 7 = 0$

17. Let $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx, m, n > 0$. If $\int_0^1 (1-x^{10})^{20}dx = a \times \beta(b,c)$, then $100(a+b+x)$ equals

(1) 1021 (2) 1120 (3) 2012 (4) 2120

18. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then $\det(AB)$ is equal to :

(1) 343 (2) 125 (3) 64 (4) 216

19. If $y(\theta) = \frac{2\cos\theta + \cos 2\theta}{\cos 3\theta + 4\cos 2\theta + 5\cos\theta + 2}$, then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to:

(1) $\frac{3}{2}$ (2) 1 (3) $\frac{1}{2}$ (4) 2

20. For $x \geq 0$, the least value of K, for which $4^{1+x} + 4^{1-x}, \frac{K}{2}, 16^x + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

(1) 10 (2) 4 (3) 8 (4) 16

SECTION-B

21. Let the mean and the standard deviation of the probability distribution

X	α	1	0	-3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma-\mu=2$, then $\sigma+\mu$ is equal to ____.

22. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{2x}{(1+x^2)^2}y=xe^{\frac{1}{(1+x^2)}}$; $y(0)=0$. Then the area enclosed by the curve $f(x)=y(x)e^{-\frac{1}{(1+x^2)}}$ and the line $y-x=4$ is ______.

23. The number of solutions of $\sin^2 x+(2+2x-x^2)\sin x-3(x-1)^2=0$, where $-\pi\le x\le\pi$, is _____.

24. Let the point $(-1,\alpha,\beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3}=\frac{y-2}{4}=\frac{z-5}{2}$ and $\frac{x+2}{-1}=\frac{y+6}{2}=\frac{z-1}{0}$. Then $(\alpha-\beta)^2$ is equal to ______.

25. If $1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\cdots$ upto $\infty=2\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a,b)=1$, then $11a+18b$ is equal to ______.

26. Let $a>0$ be a root of the equation $2x^2+x-2=0$. If $\lim_{x\to\frac{1}{a}}\frac{16(1-\cos(2+x-2x^2))}{(1-ax^2)}=\alpha+\beta\sqrt{17}$, where $\alpha,\beta\in Z$ then $\alpha+\beta$ is equal to _____.

27. If $f(t)=\int_0^{\pi}\frac{2x\,dx}{1-\cos^2 t\sin^2 x}$, $0<t<\pi$, then the value of $\int_0^{\frac{\pi}{2}}\frac{\pi^2 dt}{f(t)}$ equals _____.

28. Let the maximum and minimum values of $\left(\sqrt{8x-x^2-12}-4\right)^2+(x-7)^2, x\in R$ be M and m respectively. Then M^2-m^2 is equal to _____.

29. Let a line perpendicular to the line $2x-y=10$ touch the parabola $y^2=4(x-9)$ at the point P. The distance of the point P from the centre of the circle $x^2+y^2-14x-8y+56=0$ is _____.

30. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is _____.

NTA ANSWER

1.	(3)	2.	(3)	3.	(4)	4.	(4)	5.	(3)
6.	(2)	7.	(3)	8.	(1)	9.	(3)	10.	(3)
11.	(1)	12.	(1)	13.	(1)	14.	(4)	15.	(2)
16.	(2)	17.	(4)	18.	(4)	19.	(4)	20.	(1)
21.	(5)	22.	(18)	23.	(2)	24.	(25)	25.	(76)
26.	(170)	27.	(1)	28.	(1600)	29.	(10)	30.	(3)