JEE-MAIN EXAM APRIL, 2024

Date: - 06-04-2024 (SHIFT-2)

MATHEMATICS

SECTION-A

1. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of areas of all the triangles formed in this process, then:

(1) $\mathrm{P}^2=36\sqrt{3}\mathrm{Q}$ (2) $\mathrm{P}^2=6\sqrt{3}\mathrm{Q}$ (3) $\mathrm{P}=36\sqrt{3}\mathrm{Q}^2$ (4) $\mathrm{P}^2=72\sqrt{3}\mathrm{Q}$

2. Let $\mathrm{A}=\{1,2,3,4,5\}$. Let R be a relation on A defined by x Ry if and only if $4x\leq 5y$. Let m be the number of elements in R and n be the minimum number of elements from $\mathrm{A}\times\mathrm{A}$ that are required to be added to R to make it a symmetric relation. Then $\mathrm{m}+\mathrm{n}$ is equal to:

(1) 24 (2) 23 (3) 25 (4) 26

3. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{12}{25}$ (2) $\frac{18}{25}$ (3) $\frac{4}{25}$ (4) $\frac{6}{25}$

4. Suppose the solution of the differential equation $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2}{\beta x-2\alpha y-(\beta\gamma-4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is :

(1) $\sqrt{17}$ (2) $\frac{1}{2}$ (3) $\frac{\sqrt{17}}{2}$ (4) 2

5. If the locus of the point, whose distances from the point $(2,1)$ and $(1,3)$ are in the ratio $5:4$, is $ax^2+by^2+cxy+dx+ey+170=0$, then the value of $a^2+2b+3c+4d+e$ is equal to:

(1) 5 (2) -27 (3) 37 (4) 437

6. $\lim_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))\cdot 1}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots+n^2)}$ is equal to:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

7. Let $0\leq\mathrm{r}\leq\mathrm{n}$. If ${}^{\mathrm{n}+1}\mathrm{C}_{\mathrm{r}+1}:{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}:{}^{\mathrm{n}-1}\mathrm{C}_{\mathrm{r}-1}=55:35:21$, then $2\mathrm{n}+5\mathrm{r}$ is equal to:

(1) 60 (2) 62 (3) 50 (4) 55

8. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to :

(1) 125 (2) 150 (3) 180 (4) 160

9. If z_1, z_2 are two distinct complex number such that $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right| = 2$, then

(1) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1 .

(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1 .

(4) both z_1 and z_2 lie on the same circle.

10. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $\mathrm{x} = \frac{1}{\mathrm{e}}$ then :

(1) $\mathrm{e}^{\pi} < \pi^{\mathrm{e}}$ (2) $\mathrm{e}^{2\pi} < (2\pi)^{\mathrm{e}}$ (3) $\mathrm{e}^{\pi} > \pi^{\mathrm{e}}$ (4) $(2\mathrm{e})^{\pi} > \pi^{(2\mathrm{e})}$

11. Let $\vec{a} = 6\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$. If $\vec{c}$ is a is vector such that $|\vec{c}| \geq 6, \vec{a}.\vec{c} = 6|\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $60°$, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to:

(1) $\frac{9}{2}(6-\sqrt{6})$ (2) $\frac{3}{2}\sqrt{3}$ (3) $\frac{3}{2}\sqrt{6}$ (4) $\frac{9}{2}(6+\sqrt{6})$

12. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315^{th} position in this arrangement is :

(1) NRAGUP (2) NRAGPU (3) NRAPGU (4) NRAPUG

13. Suppose for a differentiable function $h, h(0) = 0$, $\mathrm{h}(1) = 1$ and $\mathrm{h}'(0) = \mathrm{h}'(1) = 2$. If $\mathrm{g}(\mathrm{x}) = \mathrm{h}(\mathrm{e}^{\mathrm{x}})\mathrm{e}^{\mathrm{h}(\mathrm{x})}$, then $\mathrm{g}'(0)$ is equal to:

(1) 5 (2) 3 (3) 8 (4) 4

14. Let $\mathrm{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathrm{Q}(3,-3,1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2,5,-1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14\ \mathrm{K}$, then K is equal to:

(1) 36 (2) 72 (3) 18 (4) 81

15. If $\mathrm{P}(6,1)$ be the orthocentre of the triangle whose vertices are $\mathrm{A}(5,-2)$, $\mathrm{B}(8,3)$ and $\mathrm{C}(\mathrm{h},\mathrm{k})$, then the point C lies on the circle.

(1) $x^2 + y^2 - 65 = 0$ (2) $x^2 + y^2 - 74 = 0$ (3) $x^2 + y^2 - 61 = 0$ (4) $x^2 + y^2 - 52 = 0$

16. Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R.

Then the range of the function $f(x)$ is equal to:

(1) $\left[\frac{1}{8}, \frac{1}{5}\right]$ (2) $\left[\frac{1}{7}, \frac{1}{6}\right]$ (3) $\left[\frac{1}{7}, \frac{1}{5}\right]$ (4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

17. Let $\vec{a} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \vec{b} = ((\vec{a} \times (\hat{\imath} + \hat{\jmath})) \times \hat{\imath}) \times \hat{\imath}$.

Then the square of the projection of $\vec{a}$ on $\vec{b}$ is :

(1) $\frac{1}{5}$ (2) 2 (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

18. If the area of the region

$\left\{(x,y): \frac{a}{x^2} \leq y \leq \frac{1}{x}, 1 \leq x \leq 2, 0 < a < 1\right\}$ is

$(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to:

(1) 2 (2) 0 (3) -1 (4) 1

19. If $\int \frac{1}{a^2\sin^2 x+b^2\cos^2 x}dx=\frac{1}{12}\tan^{-1}(3\tan x)+$ constant, then the maximum value of $a\sin x+b\cos x$, is :

(1) $\sqrt{40}$ (2) $\sqrt{39}$ (3) $\sqrt{42}$ (4) $\sqrt{41}$

20. If A is a square matrix of order 3 such that $\det(A)=3$ and

$\det\left(\text{adj}\left(-4\text{adj}\left(-3\text{adj}(3\text{adj}((2\ A)^{-1}))\right)\right)\right)=2^m3^n$, then $m+|\ 2n$ is equal to:

(1) 3 (2) 2 (3) 4 (4) 6

SECTION-B

21. Let $[t]$ denote the greatest integer less than or equal to t. Let $f:[0,\infty)\to R$ be a function defined by $f(x)=\left[\frac{x}{2}+3\right]-[\sqrt{x}]$. Let S be the set of all points in the interval [0,8] at which f is not continuous. Then $\sum_{a\in S} a$ is equal to

22. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x=\pm\frac{4}{\sqrt{3}}$, respectively. Let the line $y-\sqrt{3}x+\sqrt{3}=0$ touch this hyperbola at (x_0,y_0). If m is the product of the focal distances of the point (x_0,y_0), then $4e^2+m$ is equal to

23. If $S(x)=(1+x)+2(1+x)^2+3(1+x)^3+\cdots.+60(1+x)^{60}, x\neq 0$, and $(60)^2S(60)=a(b)^b+b$, where $a,b\in N$, then $(a+b)$ equal to

24. Let $[t]$ denote the largest integer less than or equal to t. If

$\int_0^3\left([x^2]+\left[\frac{x^2}{2}\right]\right)dx=a+b\sqrt{2}-\sqrt{3}-\sqrt{5}+c\sqrt{6}-\sqrt{7}$,

where $a,b,c\in z$, then $a+b+c$ is equal to

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $n-m$ is equal to

26. In a triangle $ABC, BC=7, AC=8, AB=\alpha\in N$ and $\cos A=\frac{2}{3}$. If $49\cos(3C)+42=\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is equal to

27. If the shortest distance between the lines $\frac{x-\lambda}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $\frac{x+2}{-3}=\frac{y+5}{2}=\frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to

28. Let α,β be roots of $x^2+\sqrt{2}x-8=0$.

If $U_n=\alpha^n+\beta^n$, then $\frac{U_{10}+\sqrt{12}U_9}{2U_8}$ is equal to

29. If the system of equations

$2x+7y+\lambda z=3$

$3x+2y+5z=4$

$x+\mu y+32z=-1$

has infinitely many solutions, then $(\lambda-\mu)$ is equal to

30. If the solution $y(x)$ of the given differential equation $(e^y+1)\cos x dx+e^y\sin x dy=0$ passes through the point $\left(\frac{\pi}{2},0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to

NTA ANSWER

1.	(1)	2.	(3)	3.	(1)	4.	(3)	5.	(3)
6.	(2)	7.	(3)	8.	(2)	9.	(1)	10.	(3)
11.	(4)	12.	(3)	13.	(4)	14.	(4)	15.	(1)
16.	(4)	17.	(2)	18.	(3)	19.	(1)	20.	(3)
21.	(17)	22.	(61)	23.	(3660)	24.	(23)	25.	(71)
26.	(39)	27.	(43)	28.	(4)	29.	(38)	30.	(3)