JEE-MAIN EXAM APRIL, 2024

Date: - 06-04-2024 (SHIFT-1)

MATHEMATICS

SECTION-A

1. If $f(x)=\begin{cases}x^3\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$, then

(1) $f''(0)=1$ (2) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$ (3) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$ (4) $f''(0)=0$

2. If $A(3,1,-1), B\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right), C(2,2,1)$ and $D\left(\frac{10}{3},\frac{2}{3},\frac{-1}{3}\right)$ are the vertices of a quadrilateral $ABCD$, then its area is _____.

(1) $\frac{4\sqrt{2}}{3}$ (2) $\frac{5\sqrt{2}}{3}$ (3) $2\sqrt{2}$ (4) $\frac{2\sqrt{2}}{3}$

3. $\int_0^{\pi/4}\frac{\cos^2 x\sin^2 x}{(\cos^3 x+\sin^3 x)^2}dx$ is equal to _____.

(1) $1/12$ (2) $1/9$ (3) $1/6$ (4) $1/3$

4. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On respectively, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is _____.

(1) $\sqrt{3.86}$ (2) 1.8 (3) $\sqrt{3.96}$ (4) 1.94

5. The function $f(x)=\frac{x^2+2x-15}{x^2-4x+9}, x\in R$ is _____.

(1) both one-one and onto. (2) onto but not one-one.

(3) neither one-one nor onto. (4) one-one but not onto.

6. Let $A=\{n\in[100,700]\cap N: n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is _____.

(1) 300 (2) 280 (3) 310 (4) 290

7. Let C be the circle of minimum area touching the parabola $y=6-x^2$ and the lines $y=\sqrt{3}|x|$. Then, which one of the following points lies on the circle C?

(1) $(2,4)$ (2) $(1,2)$ (3) $(2,2)$ (4) $(1,1)$

8. For $\alpha,\beta\in R$ and a natural number n, let

$A_r=\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2}\end{vmatrix}$. Then $2A_{10}-A_8$ is _____.

(1) $4\alpha+2\beta$ (2) $2\alpha+4\beta$ (3) $2n$ (4) 0

9. The shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is_____.

(1) $6\sqrt{3}$ (2) $4\sqrt{3}$ (3) $5\sqrt{3}$ (4) $8\sqrt{3}$

10. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126 p is _____.

(1) 54 (2) 64 (3) 66 (4) 56

11. Let, α,β be the distinct roots of the equation $\mathrm{x}^2-(\mathrm{t}^2-5\mathrm{t}+6)\mathrm{x}+1=0,\mathrm{t}\in\mathrm{R}$ and $\mathrm{a_n}=\alpha^{\mathrm{n}}+\beta^{\mathrm{n}}$ Then the minimum value of $\frac{\mathrm{a_{2023}}+\mathrm{a_{2025}}}{\mathrm{a_{2024}}}$ is _____.

(1) ¼ (2) $-1/2$ (3) $-1/4$ (4) $1/2$

12. Let the relations R_1 and R_2 on the set

$X=\{1,2,3,\ldots,20\}$ be given by

$\mathrm{R_1}=\{(\mathrm{x},\mathrm{y}):2\mathrm{x}-3\mathrm{y}=2\}$ and

$\mathrm{R_2}=\{(\mathrm{x},\mathrm{y}):-5\mathrm{x}+4\mathrm{y}=0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then $\mathrm{M}+\mathrm{N}$ equals _____.

(1) 8 (2) 16 (3) 12 (4) 10

13. Let a variable line of slope $m>0$ passing through the point $(4,-9)$ intersect the coordinate axes at the points A and B. the minimum value of the sum of the distances of A and B from the origin is ____.

(1) 25 (2) 30 (3) 15 (4) 10

14. The interval in which the function $\mathrm{f(x)}=\mathrm{x^x},\mathrm{x}>0$, is strictly increasing is _____.

(1) $\left(0,\frac{1}{\mathrm{e}}\right]$ (2) $\left[\frac{1}{\mathrm{e}^2},1\right)$ (3) $(0,\infty)$ (4) $\left[\frac{1}{\mathrm{e}},\infty\right)$

15. A circle in inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m+n^2$ is equal to _____.

(1) 396 (2) 408 (3) 312 (4) 414

16. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is _____.

(1) 24 (2) 56 (3) 16 (4) 48

17. Let $y=y(x)$ be the solution of the differential equation $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x},y(1)=0$. Then $y(0)$ is____.

(1) $\frac{1}{4}\left(\mathrm{e}^{\pi/2}-1\right)$ (2) $\frac{1}{2}\left(1-e^{\pi/2}\right)$ (3) $\frac{1}{4}\left(1-\mathrm{e}^{\pi/2}\right)$ (4) $\frac{1}{2}\left(\mathrm{e}^{\pi/2}-1\right)$

18. Let $\mathrm{y}=\mathrm{y(x)}$ be the solution of the differential equation $(2x\log_e x)\frac{dy}{dx}+2y=\frac{3}{x}\log_e x,x>0$ and $y(e^{-1})=0$. Then, $y(e)$ is equal to _____.

(1) $-\frac{3}{2\mathrm{e}}$ (2) $-\frac{2}{3e}$ (3) $-\frac{3}{\mathrm{e}}$ (4) $-\frac{2}{\mathrm{e}}$

19. Let the area of the region enclosed by the curves $y=3x,2y=27-3x$ and $y=3x-x\sqrt{x}$ be A. Then $10\ \mathrm{A}$ is equal to _____.

(1) 184 (2) 154 (3) 172 (4) 162

20. Let $f:(-\infty,\infty)-\{0\}\to R$ be a differentiable function such that $f'(1)=\lim_{a\to\infty}a^2f\left(\frac{1}{a}\right)$. Then $\lim_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\log_e a$ is equal to _____.

(1) $\frac{3}{2}+\frac{\pi}{4}$ (2) $\frac{3}{8}+\frac{\pi}{4}$ (3) $\frac{5}{2}+\frac{\pi}{8}$ (4) $\frac{3}{4}+\frac{\pi}{8}$

SECTION-B

21. Let $\alpha\beta\gamma=45;\alpha,\beta,\gamma\in R$. If $x(\alpha,1,2)+y(1,\beta,2)+z(2,3,\gamma)=(0,0,0)$ for some $x,y,z\in R,xyz\neq 0$, then $6\alpha+4\beta+\gamma$ is equal to_____.

22. Let a conic C pass through the point $(4,-2)$ and $P(x,y),x\geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3,-5)$. If the focal distance of the point $(7,1)$ on C is d, then 12 d equals _____.

23. Let $r_k=\frac{\int_0^1(1-x^7)^k dx}{\int_0^1(1-x^7)^{k+1}dx},k\in N$. Then the value of $\sum_{k=1}^{10}\frac{1}{7(r_k-1)}$ is equal to _____.

24. Let x_1,x_2,x_3,x_4 be the solution of the equation $4x^4+8x^3-17x^2-12x+9=0$ and $(4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2)=\frac{125}{16}m$. Then the value of m is _____.

25. Let L_1, L_2 be the lines passing through the point $P(0,1)$ and touching the parabola $9x^2+12x+18y-14=0$. Let Q and R be the points on the lines L_1 and L_2 such that the $\triangle PQR$ is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2. then $16(m_1^2+m_2^2)$ is equal to _____.

26. If the second, third and fourth terms in the expansion of $(x+y)^n$ are 135,30 and $\frac{10}{3}$, respectively, then $6(n^3+x^2+y)$ is equal to _____.

27. Let the first term of a series be $T_1=6$ and its r^{th} term $T_r=3T_{r-1}+6^r,r=2,3,\ldots\ldots,n$. If the sum of the first n terms of this series is $\frac{1}{5}(n^2-12n+39)(4.6^n-5.3^n+1)$. Then n is equal to _____.

28. For $n\in N$, if $\cot^{-1}3+\cot^{-1}4+\cot^{-1}5+\cot^{1}n=\frac{\pi}{4}$, then n is equal to_____.

29. Let P be the point $(10,-2,-1)$ and Q be the foot of the perpendicular drawn from the point $R(1,7,6)$ on the line passing through the points $(2,-5,11)$ and $(-6,7,-5)$. Then the length of the line segment PQ is equal to _____.

30. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k},\vec{b}=3\hat{i}+4\hat{j}-5\hat{k}$, and a vector $\vec{c}$ be such that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{\imath}+8\hat{\jmath}+13\hat{k}$ If $\vec{a}\cdot\vec{c}=13$, then $(24-\vec{b}\cdot\vec{c})$ is equal to _____.

NTA ANSWER

1.	**(2)**	**2.**	**(1)**	**3.**	**(3)**	**4.**	**(3)**	**5.**	**(3)**
6.	**(1)**	**7.**	**(1)**	**8.**	**(1)**	**9.**	**(2)**	**10.**	**(1)**
11.	**(3)**	**12.**	**(4)**	**13.**	**(1)**	**14.**	**(4)**	**15.**	**(2)**
16.	**(3)**	**17.**	**(2)**	**18.**	**(3)**	**19.**	**(4)**	**20.**	**(3)**
21.	**(55)**	**22.**	**(75)**	**23.**	**(65)**	**24.**	**(221)**	**25.**	**(68)**
26.	**(806)**	**27.**	**(6)**	**28.**	**(47)**	**29.**	**(13)**	**30.**	**(46)**