JEE-MAIN EXAM APRIL, 2025

Date: - 07-04-2025 (SHIFT-2)

MATHEMATICS

SECTION-A

1. Consider the lines $L_1 : x-1=y-2=z$ and $L_2 : x-2=y=z-1$. Let the feet of the perpendiculars from the point $P(5,1,-3)$ on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A , then $4A^2$ is equal to:

(1) 143 (2) 151 (3) 147 (4) 139

2. Let the system of equations

$x+5y-z=1$

$4x+3y-3z=7$

$24x+y+\lambda z=\mu$

$\lambda,\mu\in\mathbf{R}$, have infinitely many solutions. Then the number of the solutions of this system, if x, y, z are integers and satisfy $7\leqslant x+y+z\leqslant 77$, is :

(1) 3 (2) 5 (3) 6 (4) 4

3. Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be 10 . If its eccentricity is the minimum value of the function $f(t)=t^2+t+\frac{11}{12}, t\in\mathbf{R}$, then a^2+b^2 is equal to :

(1) 126 (2) 115 (3) 125 (4) 120

4. Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|}=\sqrt{2}+1$. Then $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2}$ is :

(1) $2+4\sqrt{2}$ (2) $4+2\sqrt{2}$ (3) $2+\sqrt{2}$ (4) $1+\sqrt{2}$

5. If the equation of the line passing through the point $\left(0,-\frac{1}{2},0\right)$ and perpendicular to the lines $\vec{r}=\lambda(\hat{i}+a\hat{j}+b\hat{k})$ and $\vec{r}=(\hat{i}-\hat{j}-6\hat{k})+\mu(-b\hat{i}+a\hat{j}+5\hat{k})$ is $\frac{x-1}{-2}=\frac{y+4}{d}=\frac{z-c}{-4}$, then $a+b+c+d$ is equal to :

(1) 14 (2) 13 (3) 10 (4) 12

6. Let $A=\{(\alpha,\beta)\in\mathbf{R}\times\mathbf{R}:|\alpha-1|\leqslant 4 \text{ and } |\beta-5|\leqslant 6\}$ and $B=\left\{(\alpha,\beta)\in\mathbf{R}\times\mathbf{R}:16(\alpha-2)^2+9(\beta-6)^2\leqslant 144\right\}$. Then

(1) $\mathrm{A}\cup\mathrm{B}=\{(x,y):-4\leqslant x\leqslant 4,-1\leqslant y\leqslant 11\}$ (2) $\mathrm{B}\subset\mathrm{A}$

(3) neither $\mathrm{A}\subset\mathrm{B}$ nor $\mathrm{B}\subset\mathrm{A}$ (4) $\mathrm{A}\subset\mathrm{B}$

7. Let a_n be the n^{th} term of an A.P. If $S_n=a_1+a_2+a_3+\ldots+a_n=700, a_6=7$ and $S_7=7$, then a_n is equal to:

(1) 64 (2) 70 (3) 56 (4) 65

8. If the range of the function $f(x)=\dfrac{5-x}{x^2-3x+2}, x\neq 1,2$, is $(-\infty,\alpha]\cup[\beta,\infty)$, then $\alpha^2+\beta^2$ is equal to:

(1) 192 (2) 190 (3) 194 (4) 188

9. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309 , then the sum of its first nine terms is :

(1) 755 (2) 757 (3) 760 (4) 750

10. If the area of the region $\left\{(x,y):1+x^2\leqslant y\leqslant \min\{x+7,11-3x\}\right\}$ is A , then 3 A is equal to

(1) 46 (2) 47 (3) 49 (4) 50

11. If the locus of $z\in\mathrm{C}$, such that $\operatorname{Re}\left(\dfrac{z-1}{2z+i}\right)+\operatorname{Re}\left(\dfrac{\bar{z}-1}{2\bar{z}-i}\right)=2$, is a circle of radius r and center (a,b), then $\dfrac{15ab}{r^2}$ is equal to :

(1) 24 (2) 18 (3) 16 (4) 12

12. The number of solutions of the equation $\cos 2\theta\cos\dfrac{\theta}{2}+\cos\dfrac{5\theta}{2}=2\cos^3\dfrac{5\theta}{2}$ in $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$ is :

(1) 5 (2) 7 (3) 9 (4) 6

13. If the orthocenter of the triangle formed by the lines $y=x+1, y=4x-8$ and $y=mx+c$ is at $(3,-1)$, then $\mathrm{m}-\mathrm{c}$ is :

(1) –1 (2) 4 (3) 0 (4) 2

14. The number of real roots of the equation $x|x-2|+3|x-3|+1=0$ is :

(1) 3 (2) 2 (3) 1 (4) 4

15. Let $y=y(x)$ be the solution of the differential equation $\left(x^2+1\right)y'-2xy=\left(x^4+2x^2+1\right)\cos x$, $y(0)=1$. Then $\int_{-3}^{3}y(x)\mathrm{d}x$ is :

(1) 24 (2) 30 (3) 18 (4) 36

16. Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P. If $p+q=126$, then the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{n}=1$ is :

(1) $\frac{\sqrt{7}}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{3}{4}$

17. Let $f:\mathbf{R}\to\mathbf{R}$ be a polynomial function of degree four having extreme values at $x=4$ and $x=5$. If $\lim_{x\to 0}\frac{f(x)}{x^2}=5$, then $f(2)$ is equal to :

(1) 10 (2) 8 (3) 12 (4) 14

18. Let a random variable X take values 0,1,2,3 with $P(X=0)=P(X=1)=p, P(X=2)=P(X=3)$ and $E\left(X^2\right)=2E(X)$. Then the value of $8p-1$ is:

(1) 0 (2) 2 (3) 3 (4) 1

19. A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{m}{n}, \gcd(m,n)=1$, then n^2-m^2 is equal to :

(1) 72 (2) 64 (3) 60 (4) 80

20. Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{\mathrm{b}^2}+\frac{y^2}{25}=1$ and the hyperbola $\frac{x^2}{16}-\frac{y^2}{\mathrm{b}^2}=1$, respectively. If $\mathrm{b}<5$ and $\mathrm{e}_1\mathrm{e}_2=1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is :

(1) $\frac{\sqrt{7}}{4}$ (2) $\frac{4}{5}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{3}{5}$

SECTION-B

21. For $\mathrm{t}>-1$, let α_{t} and β_{t} be the roots of the equation

$$\left((t+2)^{1/7}-1\right)x^2+\left((t+2)^{1/6}-1\right)x+\left((t+2)^{1/21}-1\right)=0. \text{ If } \lim_{t\to -1^+}\alpha_t=a \text{ and } \lim_{t\to -1^+}\beta_t=b$$

then $72(a+b)^2$ is equal to ______ .

22. Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be 2a and 2b, respectively, and one focus and the corresponding directrix of this hyperbola be (–5, 0) and 5x + 9 = 0, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p, then 4p is equal to _____ .

23. If the function $f(x)=\dfrac{\tan(\tan x)-\sin(\sin x)}{\tan x-\sin x}$ is continuous at $x=0$, then $f(0)$ is equal to_______ .

24. The sum of the series

$2\times1\times{}^{20}C_4-3\times2\times{}^{20}C_5+4\times3\times{}^{20}C_6-5\times4\times{}^{20}C_7+\cdots\cdots+18\times17\times{}^{20}C_{20}$, is equal to_______ .

25. If $\int\left(\frac{1}{x}+\frac{1}{x^3}\right)\left(\sqrt[23]{3x^{-24}+x^{-26}}\right)dx=-\frac{\alpha}{3(\alpha+1)}\left(3x^{\beta}+x^{\gamma}\right)^{\frac{\alpha+1}{\alpha}}+C, x>0, (\alpha,\beta,\gamma\in\mathbf{Z})$, where C is the constant of integration, then $\alpha+\beta+\gamma$ is equal to ______ .

NTA ANSWERS

1.	(3)	2.	(1)	3.	(1)	4.	(3)	5.	(1)	6.	(2)	7.	(1)
8.	(3)	9.	(2)	10.	(4)	11.	(2)	12.	(2)	13.	(3)	14.	(3)
15.	(1)	16.	(3)	17.	(1)	18.	(2)	19.	(4)	20.	(4)	21.	(98)
22.	(189)	23.	(2)	24.	(34)	25.	(19)						