JEE-MAIN EXAM APRIL, 2024

Date: - 08-04-2024 (SHIFT-2)

MATHEMATICS

SECTION-A

1. If the image of the point $(-4,5)$ in the line $x+2y=2$ lies on the circle $(x+4)^2+(y-3)^2=r^2$, then r is equal lo :

(1) 1 (2) 2 (3) 75 (4) 3

2. Let $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$, $\vec{b}=2\hat{\imath}+3\hat{\jmath}-5\hat{k}$ and $\vec{c}=3\hat{\imath}-\hat{\jmath}+\lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b}+\vec{c}$. If $\vec{r}\cdot\vec{a}=3$, then 3λ is equal to :

(1) 27 (2) 25 (3) 25 (4) 21

3. If $\alpha\neq a,\beta\neq b,\gamma\neq c$ and $\begin{vmatrix}\alpha & b & c\\ a & \beta & c\\ a & b & \gamma\end{vmatrix}=0$, then $\frac{a}{\alpha-a}+\frac{b}{\beta-b}+\frac{\gamma}{\gamma-c}$ is equal to :

(1) 2 (2) 3 (3) 0 (4) 1

4. In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4^{th}, 6^{th} and 8^{th} terms is :-

(1) 96 (2) 78 (3) 91 (4) 84

5. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

(1) 175 (2) 181 (3) 177 (4) 179

6. The sum of all possible values of $\theta\in[-\pi,2\pi]$, for which $\frac{1+ico\ \theta}{1-2ico\ \theta}$ is purely imaginary, is equal to

(1) 2π (2) 3π (3) 5π (4) 4π

7. If the system of equations $x+4y-z=\lambda,\ 7x+9y+\mu z=-3{,}5x+y+2z=-1$ has infinitely many solutions, then $(2\mu.+3\lambda)$ is equal to :

(1) 2 (2) -3 (3) 3 (4) -2

8. If the shortest distance between the lines $\frac{x-\lambda}{2}=\frac{y-4}{3}=\frac{z-3}{4}$ and $\frac{x-2}{4}=\frac{y-4}{6}=\frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is :

(1) $-\frac{13}{25}$ (2) $\frac{13}{25}$ (3) 1 (4) -1

9. If the value of $\frac{3\cos 36^\circ+5\sin 18^\circ}{5\cos 36^\circ-3\sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a,b,c are natural numbers and $\gcd(a,c)=1$, then $a+b+c$ is equal to :

(1) 50 (2) 40 (3) 52 (4) 54

10. Let $y=y(x)$ be the solution curve of the differential equation $\sec y\frac{dy}{dx}+2x\sin y=x^3\cos y$, $y(1)=0$. Then $y(\sqrt{3})$ is equal to :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

11. The area of the region in the first quadrant inside the circle $x^2+y^2=8$ and outside the pnrabola $y^2=2x$ is equal to :

(1) $\frac{\pi}{2}-\frac{1}{3}$ (2) $\pi-\frac{2}{3}$ (3) $\frac{\pi}{2}-\frac{2}{3}$ (4) $\pi-\frac{1}{3}$

12. If the line segment joining the points $(5,2)$ and $(2, \mathrm{a})$ subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is :

(1) 6 (2) 8 (3) 2 (4) 4

13. Let $\vec{a}=4\hat{\imath}-\hat{\jmath}+\hat{k}, \vec{b}=11\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a}+\vec{b})\times\vec{c}=\vec{c}\times(-2\vec{a}+3\vec{b})$. If $(2\vec{a}+3\vec{b})\cdot\vec{c}=1670$, then $|\vec{c}|^2$ is equal to :

(1) 1627 (2) 1618 (3) 1600 (4) 1609

14. If the function $f(x)=2x^3-9ax^2+12a^2x+1, a>0$ has a local maximum at $\mathrm{x}=\alpha$ and a local minimum $\mathrm{x}=\alpha^2$, then α and α^2 are the roots of the equation :

(1) $\mathrm{x}^2-6\mathrm{x}+8=0$ (2) $8x^2+6x-8=0$ (3) $8x^2-6x+1=0$ (4) $x^2+6x+8=0$

15. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{5}{12}$

16. Let $\int_{\alpha}^{\log_e 4}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation :

(1) $2x^2-5x+2=0$ (2) $\mathrm{x}^2-2\mathrm{x}-8=0$ (3) $2x^2-5x-2=0$ (4) $x^2+2x-8=0$

17. Let $f(\mathrm{x})=\begin{cases}-\mathrm{a} & \text{if } -\mathrm{a}\le\mathrm{x}\le 0\\ \mathrm{x}+\mathrm{a} & \text{if } 0<\mathrm{x}\le\mathrm{a}\end{cases}$

where $\mathrm{a}>0$ and $\mathrm{g}(\mathrm{x})=(f|\mathrm{x}|)-|f(\mathrm{x})|)/2$.

Then the function $\mathrm{g}:[-\mathrm{a},\mathrm{a}]\to[-\mathrm{a},\mathrm{a}]$ is

(1) neither one-one nor onto. (2) both one-one and onto.

(3) one-one. (4) onto

18. Let $\mathrm{A}=\{2,3,6,8,9,11\}$ and $\mathrm{B}=\{1,4,5,10,15\}$ Let R be a relation on $\mathrm{A}\times\mathrm{B}$ define by $(\mathrm{a},\mathrm{b})\mathrm{R}(\mathrm{c},\mathrm{d})$ if and only if $3\mathrm{ad}-7\mathrm{bc}$ is an even integer. Then the relation R is

(1) reflexive but not symmetric. (2) transitive but not symmetric.

(3) reflexive and symmetric but not transitive. (4) an equivalence relation.

19. For $a, b>0$, let

$$f(\mathrm{x})=\begin{cases}\dfrac{\tan((\mathrm{a}+1)\mathrm{x})+\mathrm{b}\tan\mathrm{x}}{\mathrm{x}}, & \mathrm{x}<0\\ 3, & \mathrm{x}=0\\ \dfrac{\sqrt{\mathrm{ax}+\mathrm{b}^2\mathrm{x}^2}-\sqrt{\mathrm{ax}}}{\mathrm{b}\sqrt{\mathrm{a}}\,\mathrm{x}\sqrt{\mathrm{x}}}, & \mathrm{x}>0\end{cases}$$

be a continous function at $x=0$. Then $\frac{\mathrm{b}}{\mathrm{a}}$ is equal to

(1) 5 (2) 4 (3) 8 (4) 6

20. If the term independent of x in the expansion of $\left(\sqrt{ax}^{\,2} + \frac{1}{2x^3}\right)^{10}$ is 105 , then a^2 is equal to :

(1) 4 (2) 9 (3) 6 (4) 2

SECTION-B

21. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is

22. If $\alpha = \lim_{x\to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right)$ and $\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then 12 $\log_e(a+b)$ is equal to

23. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S. if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to –

24. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(1,6,4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to

25. An arithmetic progression is written in the following way

			2			
		5		8		
	11		14		17	
20		23		26		29

The sum of all the terms of the 10^{th} row is

26. The number of distinct real roots of the equation $|x+1||x+3| - 4|x+2| + 5 = 0$, is

27. Let a ray of light passing through the point $(3,10)$ reflects on the line $2x + y = 6$ and the reflected ray passes through the point $(7,2)$. If the equation of the incident ray is ax + by $+1 = 0$, then $a^2 + b^2 + 3ab$ is equal to_.

28. Let $a, b, c \in N$ and $a < b < c$. Let the mean, the mean deviation about the mean and the variance of the 5 observations $9, 25, a, b, c$ be 18,4 and $\frac{136}{5}$, respectively. Then $2a + b - c$ is equal to

29. Lei $\alpha|x| = |y|e^{xy-\beta}, \alpha, \beta \in N$ be the solution of the differential equation $xdy - ydx + xy(xdy + ydx) = 0$, $y(1) = 2$. Then $\alpha + \beta$ is equal to

30. If $\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx = A\left(\frac{\alpha x - 1}{\beta x + 3}\right)^B + C$, where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is

NTA ANSWER

1.	(2)	2.	(2)	3.	(3)	4.	(3)	5.	(4)
6.	(2)	7.	(2)	8.	(3)	9.	(3)	10.	(3)
11.	(2)	12.	(4)	13.	(2)	14.	(1)	15.	(1)
16.	(1)	17.	(1)	18.	(3)	19.	(4)	20.	(1)
21.	(128)	22.	(6)	23.	(40)	24.	(11)	25.	(1505)
26.	(2)	27.	(1)	28.	(33)	29.	(4)	30.	(7)