JEE-MAIN EXAM APRIL, 2024

Date: - 08-04-2024 (SHIFT-1)

MATHEMATICS

SECTION-A

1. The value of $k \in \mathbb{N}$ for which the integral $I_n = \int_0^1 (1-x^k)^n dx, n \in \mathbb{N}$, satisfies $147I_{20} = 148I_{21}$ is :

(1) 10 (2) 8 (3) 14 (4) 7

2. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is :

(1) $1 + \log_6(8)$ (2) $\log_8(6)$ (3) $1 + \log_8(6)$ (4) $\log_8(4)$

3. Let the circles $C_1: (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2: (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point $(6,6)$. If the point $(6,6)$ divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio $2:1$, then $(\alpha+\beta) + 4(r_1^2 + r_2^2)$ equals

(1) 110 (2) 130 (3) 125 (4) 145

4. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy-plane is the point Q. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is :

(1) $\gamma\sqrt{1 - \sin^2\phi\cos^2\theta}$ (2) $\gamma\sqrt{1 + \cos^2\theta\sin^2\phi}$

(3) $\gamma\sqrt{1 - \sin^2\theta\cos^2\phi}$ (4) $\gamma\sqrt{1 + \cos^2\phi\sin^2\theta}$

5. The number of critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ is :

(1) 2 (2) 0 (3) 1 (4) 3

6. Let $f(x)$ be a positive function such that the area bounded by $y = f(x), y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is :

(1) $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (2) $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(3) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (4) $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

7. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is:

(1) 1 (2) 2 (3) 3 (4) 4

8. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3×3, then $2a + 3b$ is equal to :

(1) -10 (2) -13 (3) -9 (4) -12

9. If the shortest distance between the lines

$L_1: \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k}, \lambda \in \mathbb{R}$

$L_2: \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k}, \mu \in \mathbb{R}$

is $\frac{m}{\sqrt{n}}$, where $\gcd(m,n) = 1$, then the value of $m+n$ equals.

(1) 384 (2) 387 (3) 377 (4) 390

10. Let the sum of two positive integers be 24 . If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$, where $\gcd(m,n) = 1$, then $n-m$ equals :

(1) 9 (2) 11 (3) 8 (4) 10

11. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to :

(1) 109 (2) 108 (3) 18 (4) 19

12. Let $I(x) = \int \frac{6}{\sin^2 x(1-\text{co}\ \ x)^2} dx$. If $I(0) = 3$, then $I\left(\frac{\pi}{12}\right)$ is equal to :

(1) $\sqrt{3}$ (2) $3\sqrt{3}$ (3) $6\sqrt{3}$ (4) $2\sqrt{3}$

13. The equations of two sides AB and AC of a triangle ABC are $4x+y=14$ and $3x-2y=5$, respectively. The point $\left(2,-\frac{4}{3}\right)$ divides the third side BC internally in the ratio $2:1$. The equation of the side BC is :

(1) $x-6y-10=0$ (2) $x-3y-6=0$ (3) $x+3y+2=0$ (4) $x+6y+6=0$

14. Let $[t]$ be the greatest integer less than or equal to t. Let A be the set of al prime factors of 2310 and $f: A \rightarrow \mathbb{Z}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$.

The number of one-to-one functions from A to the range of f is :

(1) 20 (2) 120 (3) 25 (4) 24

15. Let z be a complex number such that $|z+2| = 1$ and $\text{lm}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $|\text{Re}(\overline{z+2})|$ is :

(1) $\frac{\sqrt{6}}{5}$ (2) $\frac{1+\sqrt{6}}{5}$ (3) $\frac{24}{5}$ (4) $\frac{2\sqrt{6}}{5}$

16. If the set $R = \{(a,b); a+5b=42, a,b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^{m}(1+i^{n!}) = x+iy$, where $I = \sqrt{-1}$, then the value of $m+x+y$ is :

(1) 8 (2) 12 (3) 4 (4) 5

17. For the function $f(x) = (\cos x) - x + 1, x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x is $[0,\pi]$.

(S2) $f(x)$ is decreasing in $\left[0,\frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2},\pi\right]$.

(1) Both (S1) and (S2) are correct (2) Only (S1) is correct

(3) Both (S1) and (S2) are incorrect (4) Only (S2) is correct

18. The set of all α, for which the vector $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$ is :

(1) $[0,1)$ (2) $(-2,0]$ (3) $\left(-\frac{4}{3},0\right]$ (4) $\left(-\frac{4}{3},1\right)$

19. Let $y = y(x)$ be the solution of the differential equation $(1+y^2)e^{\tan x}dx + \cos^2 x(1+e^{2\tan x})dy = 0, y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{2}{e}$ (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) $\frac{2}{e^2}$

20. Let $\mathrm{H}: \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6), \alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to :

(1) 170 (2) 171 (3) 169 (4) 172

SECTION-B

21. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^{n}, then n is equal to

22. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0, x + 2y - 1 = 0$ and ax + by $-1 = 0$, is the centroid of another triangle, whose circumecentre and orthocentre respectively are $(3,4)$ and $(-6,-8)$, then the value of $|a-b|$ is

23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to

24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3 , is equal to

25. Let the positive integers be written in the form :

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \\ \vdots & & & & & & \vdots \end{array}$$

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is____.

26. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}, \theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to

27. Let $\alpha = \sum_{\mathrm{r}=0}^{\mathrm{n}}(4\mathrm{r}^2 + 2\mathrm{r} + 1)^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}$ and $\beta = \left(\sum_{r=0}^{n} \frac{{}^{n}C_r}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is

28. Let $\vec{a} = 9\hat{\imath} - 13\hat{\jmath} + 25\hat{k}, \vec{b} = 3\hat{\imath} + 7\hat{\jmath} - 13\hat{k}$ and $\vec{c} = 17\hat{\imath} - 2\hat{\jmath} + \hat{k}$ be three given vectros. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to

29. Let the area of the region enclosed by the curve $\mathrm{y} = \min\{\sin \mathrm{x}, \cos \mathrm{x}\}$ and the x-axis between $\mathrm{x} = -\pi$ to $\mathrm{x} = \pi$ be A. Then A^2 is equal to

30. The value of $\lim_{x \to 0} 2 \left(\frac{1 - \cos x \sqrt{\cos 2x} \sqrt[3]{\cos 3x} \ldots\ldots 10}{x^2} \right)$ is

NTA ANSWER

1.	**(4)**	**2.**	**(3)**	**3.**	**(2)**	**4.**	**(1)**	**5.**	**(1)**
6.	**(3)**	**7.**	**(2)**	**8.**	**(2)**	**9.**	**(2)**	**10.**	**(4)**
11.	**(1)**	**12.**	**(2)**	**13.**	**(3)**	**14.**	**(2)**	**15.**	**(4)**
16.	**(2)**	**17.**	**(2)**	**18.**	**(3)**	**19.**	**(3)**	**20.**	**(2)**
21.	**(7)**	**22.**	**(16)**	**23.**	**(17)**	**24.**	**(36)**	**25.**	**(103)**
26.	**(96)**	**27.**	**(5)**	**28.**	**(569)**	**29.**	**(16)**	**30.**	**(55)**