JEE-MAIN EXAM APRIL, 2024

Date: - 09-04-2024 (SHIFT-2)

MATHEMATICS

SECTION-A

1. $\text{Lim}_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

(1) e (2) $\frac{-2}{e}$ (3) 0 (4) e-e e^2

2. Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3},\frac{11}{3},\frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2}=\frac{3y-11}{1}=\frac{3z-19}{2}$ is equal to :

(1) 3 (2) 5 (3) 4 (4) 6

3. Let $\int_0^x\sqrt{1-(y'(t))^2}dt=\int_0^x y(t)dt, 0\le x\le 3, y\ge 0, y(0)=0$. Then at $x=2, y''+y+1$ is equal to :

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) $1/2$

4. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to :

(1) 12 (2) ∞ (3) 10 (4) 8

5. The area (in square units) of the region enclosed by the ellipse $x^2+3y^2=18$ in the first quadrant below the line $y=x$ is :

(1) $\sqrt{3}\pi+\frac{3}{4}$ (2) $\sqrt{3}\pi$ (3) $\sqrt{3}\pi-\frac{3}{4}$ (4) $\sqrt{3}\pi+1$

6. Let the foci of a hyperbola H coincide with the foci of the ellipse $E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2+2\beta^2$ is equal to :

(1) 242 (2) 225 (3) 237 (4) 205

7. Two vertices of a triangle ABC are $A(3,-1)$ and $B(-2,3)$, and its orthocentre is $P(1,1)$. If the coordinates of the point C are (α,β) and the centre of the circle circumscribing the triangle PAB is (h,k), then the value of $(\alpha+\beta)+2(h+k)$ equals :

(1) 51 (2) 81 (3) 5 (4) 15

8. If the variance of the frequency distribution is 160, then the value of $c\in N$ is

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

(1) 5 (2) 8 (3) 7 (4) 6

9. Let the range of the function $f(x)=\frac{1}{2+\sin 3x+\cos 3x}, x \in \mathrm{IR}$ be $[a, b]$. If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to :

(1) $\sqrt{2}$ (2) 2 (3) $\sqrt{\pi}$ (4) π

10. Between the following two statements :

Statement-I : Let $\vec{a}=\hat{\mathrm{i}}+2\hat{\mathrm{j}}-3\hat{\mathrm{k}}$ and $\vec{b}=2\hat{\imath}+\hat{\jmath}-\hat{k}$. Then the vector $\vec{\mathrm{r}}$ satisfying $\vec{a}\times\vec{r}=\vec{a}\times\vec{b}$ and $\vec{a}\cdot\vec{r}=0$ is of magnitude $\sqrt{10}$.

Statement-II : In a triangle $\mathrm{ABC}, \cos 2\,\mathrm{A}+\cos 2\,\mathrm{B}+\cos 2\mathrm{C} \geq -\frac{3}{2}$.

(1) Both Statement-I and Statement-II are incorrect

(2) Statement-I is incorrect but Statement-II is correct

(3) Both Statement-I and Statement-II are correct

(4) Statement-I is correct but Statement-II is incorrect

11. $\lim_{x\to\frac{x}{2}}\left(\frac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to :

(1) $\frac{9\pi^2}{8}$ (2) $\frac{11\pi^2}{10}$ (3) $\frac{3\pi^2}{2}$ (4) $\frac{5\pi^2}{9}$

12. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is :

(1) $21/4$ (2) $69/16$ (3) $63/16$ (4) $19/4$

13. Let $B=\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and A be a 2×2 matrix such that $\mathrm{AB}^{-1}=\mathrm{A}^{-1}$. If $\mathrm{BCB}^{-1}=\mathrm{A}$ and $\mathrm{C}^4+\alpha\mathrm{C}^2+\beta\mathrm{I}=\mathrm{O}$, then $2\beta-\alpha$ is equal to :

(1) 16 (2) 2 (3) 8 (4) 10

14. If $\log_e y=3\sin^{-1}x$, then $(1-x)^2y''-xy'$ at $x=\frac{1}{2}$ is equal to :

(1) $9\mathrm{e}^{\pi/6}$ (2) $3\mathrm{e}^{\pi/6}$ (3) $3\mathrm{e}^{\pi/2}$ (4) $9\mathrm{e}^{\pi/2}$

15. The integral $\int_{1/4}^{3/4}\cos\left(2\cot^{-1}\sqrt{\frac{1-\mathrm{x}}{1+\mathrm{x}}}\right)\mathrm{dx}$ is equal to:

(1) $-1/2$ (2) ¼ (3) ½ (4) $-1/4$

16. Let a, ar, ar^{2^2}.....be an infinite G.P. If $\sum_{\mathrm{n}=0}^{\infty}\mathrm{ar}^{\mathrm{n}}=57$ and $\sum_{\mathrm{n}=0}^{\infty}\mathrm{a}^3\mathrm{r}^{3\mathrm{n}}=9747$, then $\mathrm{a}+18\mathrm{r}$ is equal to :

(1) 27 (2) 46 (3) 38 (4) 31

17. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{\mathrm{th}}$ roll, $\mathrm{i}=2,3$, is equal to :

(1) $3/54$ (2) $2/54$ (3) $5/54$ (4) $1/54$

18. The value of the integral $\int_{-1}^{2}\log_e\left(x+\sqrt{x^2+1}\right)dx$ is :

(1) $\sqrt{5}-\sqrt{2}+\log_{\mathrm{e}}\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$ (2) $\sqrt{2}-\sqrt{5}+\log_{\mathrm{e}}\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{5}-\sqrt{2}+\log_{\mathrm{e}}\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$ (4) $\sqrt{2}-\sqrt{5}+\log_{\mathrm{e}}\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

19. Let $\alpha,\beta;\alpha>\beta$, be the roots of the equation $x^2-\sqrt{2}x-\sqrt{3}=0$. Let $P_n=\alpha^n-\beta^n, n\in N$. Then $(11\sqrt{3}-10\sqrt{2})\mathrm{P}_{10}+(11\sqrt{2}+10)\mathrm{P}_{11}-11\mathrm{P}_{12}$ is equal to :

(1) $10\sqrt{2}\mathrm{P}_9$ (2) $10\sqrt{3}\mathrm{P}_9$ (3) $11\sqrt{2}\mathrm{P}_9$ (4) $11\sqrt{3}\mathrm{P}_9$

20. Let $\vec{a} = 2\hat{\imath} + \alpha\hat{\jmath} + \hat{k}$, $\vec{b} = -\hat{\imath} + \hat{k}$, $\vec{c} = \beta\hat{\jmath} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2). Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to

(1) 17 (2) 24 (3) 21 (4) 19

SECTION-B

21. Consider the circle $C: x^2 + y^2 = 4$ and the parabola $P: y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to

22. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then $16ab$ is equal to ____.

23. For a differentiable function $f: IR \to IR$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in IR, f(0) = 1$ and $\lim_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to

24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is

25. Let $A = \{(x, y): 2x + 3y = 23, x, y \in N\}$ and $B = \{x: (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to

26. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to _____.

27. The square of the distance of the image of the point $(6,1,5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is

28. If $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \cdots + \frac{1}{\alpha+101}\right) - \left(\frac{1}{2 \cdot 1} + \frac{1}{4 \cdot 3} + \frac{1}{6 \cdot 5} + \cdots + \frac{1}{2024 \cdot 2023}\right) = \frac{1}{2024}$, then α is equal to-

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1} x + 3\cos^{-1} x = \frac{2\pi}{5}$, is

30. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations $AX = B$ has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b). Then $8\int_a^b |A| dm$ is equal to

NTA ANSWER

1.	**(1)**	**2.**	**(1)**	**3.**	**(1)**	**4.**	**(1)**	**5.**	**(2)**
6.	**(2)**	**7.**	**(3)**	**8.**	**(3)**	**9.**	**(1)**	**10.**	**(2)**
11.	**(1)**	**12.**	**(1)**	**13.**	**(4)**	**14.**	**(4)**	**15.**	**(4)**
16.	**(4)**	**17.**	**(3)**	**18.**	**(2)**	**19.**	**(2)**	**20.**	**(4)**
21.	**(80)**	**22.**	**(252)**	**23.**	**(61)**	**24.**	**(70)**	**25.**	**(24)**
26.	**(36)**	**27.**	**(62)**	**28.**	**(1011)**	**29.**	**(0)**	**30.**	**(450)**