JEE-MAIN EXAM APRIL, 2024

Date: - 09-04-2024 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Let the line L intersect the lines $x-2=-y=z-1, 2(x+1)=2(y-1)=z+1$ and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L ?

(1) $\left(-\frac{1}{3},1,1\right)$ (2) $\left(-\frac{1}{3},1,-1\right)$ (3) $\left(-\frac{1}{3},-1,-1\right)$ (4) $\left(-\frac{1}{3},-1,1\right)$

2. The parabola $y^2=4x$ divides the area of the circle $x^2+y^2=5$ in two parts. The area of the smaller part is equal to :

(1) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (3) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

3. The solution curve, of the differential equation $2y\frac{dy}{dx}+3=5\frac{dy}{dx}$, passing through the point $(0,1)$ is a conic, whose vertex lies on the line :

(1) $2x+3y=9$ (2) $2x+3y=-9$ (3) $2x+3y=-6$ (4) $2x+3y=6$

4. A ray of light coming from the point $P(1,2)$ gets reflected from the point Q on the x-axis and then passes through the point $R(4,3)$. If the point $S(h,k)$ is such that PQRS is a parallelogram, then hk^2 is equal to :

(1) 80 (2) 90 (3) 60 (4) 70

5. Let $\lambda,\mu\in R$. If the system of equations

$3x+5y+\lambda z=3$

$7x+11y-9z=2$

$97x+155y-189z=\mu$

has infinitely many solutions, then $\mu+2\lambda$ is equal to :

(1) 25 (2) 24 (3) 27 (4) 22

6. The coefficient of x^{70} in $x^2(1+x)^{98}+x^3(1+x)^{97}+x^4(1+x)^{96}++x^{54}(1+x)^{46}$ is ${}^{99}C_p-{}^{46}C_q$.

Then a possible value to $p+q$ is :

(1) 55 (2) 61 (3) 68 (4) 83

7. Let $\int\frac{2-\tan x}{3+\tan x}dx=\frac{1}{2}(\alpha x+\log_e|\beta\sin x+\gamma\cos x|)+C$, where C is the constant of integration. Then $\alpha+\frac{\gamma}{\beta}$ is equal to :

(1) 3 (2) 1 (3) 4 (4) 7

8. A variable line L passes through the point $(3,5)$ and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

(1) 30 (2) 25 (3) 40 (4) 35

9. Let $|\cos\theta\cos(60-\theta)\cos(60-\theta)| \leq \frac{1}{8}, \theta \in [0,2\pi]$ Then, the sum of all $\theta \in [0,2\pi]$, where $\cos 3\theta$ attains its maximum value, is :

(1) 9π (2) 18π (3) 6π (4) 15π

10. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral $OABC$ is equal to :

(1) 38 (2) 40 (3) 32 (4) 35

11. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R-(\alpha,\beta)$ then $12\alpha\beta$ is equal to :

(1) 36 (2) 24 (3) 40 (4) 32

12. If the sum of series $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2\,d)} + \cdots \ldots + \frac{1}{(1+9\,d)(1+10\,d)}$ is equal to 5 , then $50\,d$ is equal to :

(1) 20 (2) 5 (3) 15 (4) 10

13. Let $f(x) = ax^3 + bx^2 + ex + 41$ be such that $f(1) = 40, f'(1) = 2$ and $f''(1) = 4$. Then $a^2 + b^2 + c^2$ is equal to :

(1) 62 (2) 73 (3) 54 (4) 51

14. Let a circle passing through $(2,0)$ have its centre at the point (h,k). Let (x_c, y_c) be the point of intersection of the lines $3x + 5y = 1$ and $(2+c)x + 5c^2y = 1$. If $h = \lim_{c\to 1} x_c$ and $k = \lim_{c\to 1} y_c$, then the equation of the circle is :

(1) $25x^2 + 25y^2 - 20x + 2y - 60 = 0$ (2) $5x^2 + 5y^2 - 4x - 2y - 12 = 0$

(3) $25x^2 + 25y^2 - 2x + 2y - 60 = 0$ (4) $5x^2 + 5y^2 - 4x + 2y - 12 = 0$

15. The shortest distance between the line $\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5}$ and $\frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$ is :

(1) $\frac{187}{\sqrt{563}}$ (2) $\frac{178}{\sqrt{563}}$ (3) $\frac{185}{\sqrt{563}}$ (4) $\frac{179}{\sqrt{563}}$

16. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25 , then $4x + 5y$ is equal to :

(1) 43 (2) 44 (3) 47 (4) 46

17. The solution of the differential equation $(x^2 + y^2)dx - 5xydy = 0, y(1) = 0$, is :

(1) $|x^2 - 4y^2|^5 = x^2$ (2) $|x^2 - 2y^2|^6 = x$ (3) $|x^2 - 4y^2|^6 = x$ (4) $|x^2 - 2y^2|^5 = x^2$

18. Let three vectors $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}, \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ from a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. if α is a positive real number, then $|\vec{c}|^2$ is :

(1) 16 (2) 14 (3) 12 (4) 10

19. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is :

(1) $x^2 - 190x + 9466 = 0$ (2) $x^2 - 195x + 9466 = 0$

(3) $x^2 - 195x + 9506 = 0$ (4) $x^2 - 180x + 9506 = 0$

20. Let $f(x)=x^2+9, g(x)=\frac{x}{x-9}$ and $a=\text{fog}(10), b=\text{gof}(3)$. If e and 1 denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a}+\frac{y^2}{b}=1$, then $8e^2+1^2$ is equal to.

(1) 16 (2) 8 (3) 6 (4) 12

SECTION-B

21. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked $1,2,3,4$. If the probability that $ax^2+bx+c=0$ has all real roots is $\frac{m}{n}$, $\gcd(\mathrm{m},\mathrm{n})=1$, then $\mathrm{m}+\mathrm{n}$ is equal to

22. The sum of the square of the modulus of the elements in the set $\{\mathrm{z}=\mathrm{a}+\mathrm{ib}: \mathrm{a}, \mathrm{b} \in \mathrm{Z}, \mathrm{z} \in \mathrm{C}, |\mathrm{z}-1| \leq 1, |\mathrm{z}-5| \leq |\mathrm{z}-5\mathrm{i}|\}$ is

23. Let the set of all positive values of λ, for which the point of local minimum of the function $\left(1+x(\lambda^2-x^2)\right)$ satisfies $\frac{x^2+x+2}{x^2+5x+6}<0$, be (α, β). Then $\alpha^2+\beta^2$ is equal to

24. Let

$$\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{(n^2+1)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}-\frac{8n}{(n^2+4)\sqrt{n^4+16}}\right.$$
$$\left.+\cdots\ldots+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right) \text{ be } \frac{\pi}{k},$$

using only the principal values of the inverse trigonometric functions. Then k^2 is equal to

25. The remainder when 428^{2024} is divided by 21 is

26. Lef $\mathrm{f}:(0,\pi)\to \mathrm{R}$ be a function given by

$$f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$$

Where $\mathrm{a}, \mathrm{b} \in \mathrm{Z}$. If f is continuous at $\mathrm{x}=\frac{\pi}{2}$, then $\mathrm{a}^2+\mathrm{b}^2$ is equal to

27. Let A be a non-singular matrix of order 3. If $\det(3\operatorname{adj}(2\operatorname{adj}((\det A)A)))=3^{-13}\cdot 2^{-1}$ and $\det(3\operatorname{adj}(2\mathrm{A}))=2^{\mathrm{m}}\cdot 3^{\mathrm{n}}$, then $|3\mathrm{m}+2\mathrm{n}|$ is equal to

28. Let the centre of a circle, passing through the point $(0,0),(1,0)$ and touching the circle $x^2+y^2=9$, be (h,k). Then for all possible values of the coordinates of the centre $(\mathrm{h},\mathrm{k}), 4(\mathrm{h}^2+\mathrm{k}^2)$ is equal to

29. If a function f satisfies $f(m+n)=f(m)+f(n)$ for all $\mathrm{m},\mathrm{n} \in \mathrm{N}$ and $\mathrm{f}(1)=1$, then the largest natural number λ such that $\sum_{\mathrm{k}=1}^{2022}\mathrm{f}(\lambda+\mathrm{k}) \leq (2022)^2$ is equal to

30. Let $\mathrm{A}=\{2,3,6,7\}$ and $\mathrm{B}=\{4,5,6,8\}$. Let R be a relation defined on $A\times B$ by $(a_1,b_1)R(a_2,b_2)$ is and only if $a_1+a_2=b_1+b_2$. Then the number of elements in R is

NTA ANSWER

1.	(2)	2.	(1)	3.	(1)	4.	(4)	5.	(1)
6.	(4)	7.	(3)	8.	(1)	9.	(3)	10.	(4)
11.	(4)	12.	(2)	13.	(4)	14.	(1)	15.	(1)
16.	(2)	17.	(1)	18.	(2)	19.	(3)	20.	(2)
21.	(19)	22.	(9)	23.	(39)	24.	(32)	25.	(1)
26.	(81)	27.	(14)	28.	(9)	29.	(1010)	30.	(25)