JEE-MAIN EXAM JANUARY, 2025

Date: - 22-01-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Let for $f(x)=7\tan^8 x+7\tan^6 x-3\tan^4 x-3\tan^2 x$, $I_1=\int_0^{\pi/4} f(x)dx$ and $I_2=\int_0^{\pi/4} xf(x)dx$. Then $7I_1+12I_2$ is equal to :

(1) π (2) 2 (3) 1 (4) 2π

2. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is equal to :

(1) 4 (2) 14 (3) 13 (4) 11

3. Let the foci of a hyperbola be $(1,14)$ and $(1,-12)$. If it passes through the point $(1,6)$, then the length of its latus-rectum is :

(1) $\frac{25}{6}$ (2) $\frac{288}{5}$ (3) $\frac{144}{5}$ (4) $\frac{24}{5}$

4. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point $(2,5)$ and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α,β), then $3\beta-2\alpha$ is equal to :

(1) 12 (2) 14 (3) 15 (4) 10

5. The product of all solutions of the equation $e^{5(\log_e x)^2+3}=x^8, x>0$, is :

(1) e^2 (2) e (3) $e^{8/5}$ (4) $e^{6/5}$

6. If $\sum_{r=1}^{n} T_r=\frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{T_r}\right)$ is equal to :

(1) $\frac{1}{3}$ (2) 1 (3) 0 (4) $\frac{2}{3}$

7. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64\left(\mu+\sigma^2\right)$ is :

(1) 64 (2) 51 (3) 48 (4) 32

8. Let $f:\mathbf{R}\to\mathbf{R}$ be a twice differentiable function such that $f(x+y)=f(x)f(y)$ for all $x,y\in\mathbf{R}$. If $f'(0)=4a$ and f satisfies $f''(x)-3af'(x)-f(x)=0, a>0$, then the area of the region $R=\{(x,y)|\ 0\le y\le f(ax), 0\le x\le 2\}$ is:

(1) e^4-1 (2) e^4+1 (3) e^2-1 (4) e^2+1

9. Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16\left(\left(\sec^{-1}x\right)^2+\left(\text{cosec}^{-1}x\right)^2\right)$ is :

(1) $24\pi^2$ (2) $22\pi^2$ (3) $18\pi^2$ (4) $31\pi^2$

10. Let $A=\{1,2,3,\ldots,10\}$ and $B=\left\{\frac{m}{n}: m,n\in A, m<n \text{ and } gcd(m,n)=1\right\}$. Then $n(B)$ is equal to :

(1) 36 (2) 37 (3) 31 (4) 29

11. The area of the region, inside the circle $(x-2\sqrt{3})^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is:

(1) $6\pi-8$ (2) $3\pi-8$ (3) $6\pi-16$ (4) $3\pi+8$

12. Let $x=x(y)$ be the solution of the differential equation $y^2\,dx+\left(x-\frac{1}{y}\right)dy=0$. If $x(1)=1$, then $x\left(\frac{1}{2}\right)$ is :

(1) $3-e$ (2) $3+e$

(3) $\frac{3}{2}+e$ (4) $\frac{1}{2}+e$

13. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z|=1$ with $\arg(z_1)=\frac{-\pi}{4}, \arg(z_2)=0$ and $\arg(z_3)=\frac{\pi}{4}$. If $\left|z_1\bar{z}_2+z_2\bar{z}_3+z_3\bar{z}_1\right|^2=\alpha+\beta\sqrt{2}, \alpha,\beta\in\mathbf{Z}$, then the value of $\alpha^2+\beta^2$ is:

(1) 24 (2) 29 (3) 31 (4) 41

14. Let $f(x)$ be a real differentiable function such that $f(0)=1$ and $f(x+y)=f(x)f'(y)+f'(x)f(y)$ for all $x,y\in\mathbf{R}$. Then $\sum_{n=1}^{100}\log_e f(n)$ is equal to :

(1) 5220 (2) 2406 (3) 2384 (4) 2525

15. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1a_5=28$ and $a_2+a_4=29$, then a_6 is equal to :

(1) 784 (2) 812 (3) 628 (4) 526

16. The number of non-empty equivalence relations on the set {1,2,3} is :

(1) 4 (2) 6 (3) 7 (4) 5

17. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

(1) $\left(-\frac{5}{3}, -7, 1\right)$ (2) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$ (3) $\left(2, 3, \frac{1}{3}\right)$ (4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

18. Let the triangle PQR be the image of the triangle with vertices $(1,3), (3,1)$ and $(2,4)$ in the line $x+2y=2$. If the centroid of ΔPQR is the point (α, β), then $15(\alpha-\beta)$ is equal to :

(1) 21 (2) 22 (3) 24 (4) 19

19. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R . If the circle C with centre at $(-1,-1)$ passes through the points P, Q and R, then the area of ΔPQR is :

(1) 5 (2) 6 (3) 7 (4) 4

20. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is ' M ', is :

(1) 14950 (2) 5148 (3) 4356 (4) 6084

SECTION-B

21. Let the function,

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geqslant 1 \end{cases}$$

be differentiable for all $x \in \mathbf{R}$, where $a > 1, b \in \mathbf{R}$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha + \beta\sqrt{3}, \alpha, \beta \in Z$, then the value of $\alpha + \beta$ is ___ .

22. Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}, \lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is ____ .

23. If $\sum_{r=0}^{5} \frac{^{11}C_{2r+1}}{2r+2} = \frac{m}{n}, \gcd(m,n) = 1$, then $m - n$ is equal to _____.

24. Let A be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\,\text{adj}(-6\,\text{adj}(3A))) = 2^{m+n} \cdot 3^{mn}, m > n$. Then $4m + 2n$ is equal to ____ .

25. Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}, \alpha \in \mathbf{R}$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point $A(1,1,-1)$ on L_2, then the value of $26\alpha(PB)^2$ is _____ .

NTA ANSWERS

1.	**(3)**	**2.**	**(2)**	**3.**	**(2)**	**4.**	**(3)**	**5.**	**(3)**	**6.**	**(4)**	**7.**	**(3)**
8.	**(3)**	**9.**	**(2)**	**10.**	**(3)**	**11.**	**(3)**	**12.**	**(1)**	**13.**	**(2)**	**14.**	**(4)**
15.	**(1)**	**16.**	**(4)**	**17.**	**(4)**	**18.**	**(2)**	**19.**	**(2)**	**20.**	**(2)**	**21.**	**34**
22.	**16**	**23.**	**2035**	**24.**	**34**	**25.**	**216**						