JEE-MAIN EXAM JANUARY, 2025

Date: - 22-01-2025 (SHIFT-2)

MATHEMATICS

SECTION-A

1. If A and B are two events such that $P(A\cap B)=0.1$, and $P(A\,|\,B)$ and $P(B|\,A)$ are the roots of the equation $12x^2-7x+1=0$, then the value of $\frac{P(\bar{A}\cup\bar{B})}{P(\bar{A}\cap\bar{B})}$ is :

(1) $\frac{4}{3}$ (2) $\frac{7}{4}$ (3) $\frac{5}{3}$ (4) $\frac{9}{4}$

2. If $\lim\limits_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e}-\frac{x}{1+x}\right)\right)^x=\alpha$, then the value of $\frac{\log_e\alpha}{1+\log_e\alpha}$ equals :

(1) e (2) e^{-2} (3) e^2 (4) e^{-1}

3. Let $P(4,4\sqrt{3})$ be a point on the parabola $y^2=4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to :

(1) $\frac{263\sqrt{3}}{8}$ (2) $17\sqrt{3}$ (3) $\frac{34\sqrt{3}}{3}$ (4) $\frac{343\sqrt{3}}{8}$

4. Let α,β,γ and δ be the coefficients of x^7,x^5,x^3 and x respectively in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, x>1$. If u and v satisfy the equations $\alpha u+\beta v=18$, $\gamma u+\delta v=20$, then $u+v$ equals :

(1) 8 (2) 3 (3) 5 (4) 4

5. Let $f(x)=\int_0^{x^2}\frac{t^2-8t+15}{e^t}dt, x\in\mathbf{R}$. Then the numbers of local maximum and local minimum points of f, respectively, are :

(1) 1 and 3 (2) 3 and 2 (3) 2 and 2 (4) 2 and 3

6. Let $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ and $H:\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If $a-A=2$, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to :

(1) 8 (2) 10 (3) 9 (4) 7

7. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a}+2\vec{b}$ and $3\vec{a}-\lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1,3]$ is :

(1) 3 (2) 1 (3) 0 (4) 2

8. The sum of all values of $\theta\in[0,2\pi]$ satisfying $2\sin^2\theta=\cos 2\theta$ and $2\cos^2\theta=3\sin\theta$ is

(1) $\frac{\pi}{2}$ (2) $\frac{5\pi}{6}$ (3) π (4) 4π

9. If $\int e^x\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\frac{\sin^{-1}x}{\left(1-x^2\right)^{3/2}}+\frac{x}{1-x^2}\right)dx=g(x)+C$, where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals :

(1) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$ (2) $\frac{\pi}{6}\sqrt{\frac{e}{2}}$

(3) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$ (4) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$

10. For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A|=\frac{1}{2}$ and trace $(A)=3$. If $B=\text{adj}(\text{adj}(2A))$, then the value of $|B|+$ trace (B) equals :

(1) 280 (2) 132 (3) 56 (4) 174

11. Let a line pass through two distinct points $P(-2,-1,3)$ and Q, and be parallel to the vector $3\hat{i}+2\hat{j}+2\hat{k}$. If the distance of the point Q from the point $R(1,3,3)$ is 5, then the square of the area of ΔPQR is equal to :

(1) 144 (2) 136 (3) 140 (4) 148

12. The area of the region enclosed by the curves $y=x^2-4x+4$ and $y^2=16-8x$ is :

(1) 5 (2) $\frac{8}{3}$

(3) 8 (4) $\frac{4}{3}$

13. In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

(1) 120 (2) 72 (3) 144 (4) 96

14. If the system of linear equations :

$x+y+2z=6$

$2x+3y+az=a+1$

$-x-3y+bz=2b$

where $a,b\in\mathbf{R}$, has infinitely many solutions, then $7a+3b$ is equal to :

(1) 9 (2) 16 (3) 22 (4) 12

15. If $x=f(y)$ is the solution of the differential equation

$$\left(1+y^2\right)+\left(x-2e^{\tan^{-1}y}\right)\frac{dy}{dx}=0, y\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

with $f(0)=1$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(1) $e^{\pi/12}$ (2) $e^{\pi/6}$ (3) $e^{\pi/3}$ (4) $e^{\pi/4}$

16. Let α_θ and β_θ be the distinct roots of $2x^2+(\cos\theta)x-1=0, \theta\in(0,2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4+\beta_\theta^4$, then $16(M+m)$ equals :

(1) 27 (2) 17 (3) 24 (4) 25

17. Suppose that the number of terms in an A.P. is $2k, k\in\mathbf{N}$. If the sum of all odd terms of the A.P. is 40 , the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27 , then k is equal to :

(1) 4 (2) 5 (3) 6 (4) 8

18. Let $A=\{1,2,3,4\}$ and $B=\{1,4,9,16\}$. Then the number of many-one functions $f:A\to B$ such that $1\in f(A)$ is equal to :

(1) 127 (2) 163 (3) 139 (4) 151

19. Let the curve $z(1+i)+\bar{z}(1-i)=4, z\in\mathbf{C}$, divide the region $|z-3|\le 1$ into two parts of areas α and β. Then $|\alpha-\beta|$ equals :

(1) $1+\frac{\pi}{4}$ (2) $1+\frac{\pi}{2}$ (3) $1+\frac{\pi}{3}$ (4) $1+\frac{\pi}{6}$

20. The perpendicular distance, of the line $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z+3}{2}$ from the point $P(2,-10,1)$, is :

(1) $5\sqrt{2}$ (2) $4\sqrt{3}$ (3) $3\sqrt{5}$ (4) 6

SECTION-B

21. Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _____ .

22. Let $A(6,8), B(10\cos\alpha, -10\sin\alpha)$ and $C(-10\sin\alpha, 10\cos\alpha)$, be the vertices of a triangle. If $L(a,9)$ and $G(h,k)$ be its orthocenter and centroid respectively, then $(5a-3h+6k+100\sin 2\alpha)$ is equal to _____.

23. If $\sum_{r=1}^{30} \frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}} = \alpha \times 2^{29}$, then α is equal to

24. Let $A=\{1,2,3\}$. The number of relations on A, containing $(1,2)$ and $(2,3)$, which are reflexive and transitive but not symmetric, is _____.

25. Let $y=f(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^6+4x}{\sqrt{1-x^2}}, -1<x<1$ such that $f(0)=0$. If $6\int_{-1/2}^{1/2} f(x)dx = 2\pi-\alpha$ then α^2 is equal to _____.

NTA ANSWERS

1.	**(4)**	**2.**	**(1)**	**3.**	**(4)**	**4.**	**(3)**	**5.**	**(4)**	**6.**	**(1)**	**7.**	**(3)**
8.	**(3)**	**9.**	**(4)**	**10.**	**(1)**	**11.**	**(2)**	**12.**	**(2)**	**13.**	**(3)**	**14.**	**(2)**
15.	**(2)**	**16.**	**(4)**	**17.**	**(2)**	**18.**	**(4)**	**19.**	**(2)**	**20.**	**(3)**	**21.**	**28**
22.	**145**	**23.**	**465**	**24.**	**3**	**25.**	**27**						