JEE-MAIN EXAM JANUARY, 2025

Date: - 23-01-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Let $f(x)=\log_e x$ and $g(x)=\dfrac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$. Then the domain of fog is

(1) $\mathbb{R}$ (2) $[1,\infty)$ (3) $(0,\infty)$ (4) $[0,\infty)$

2. If $\dfrac{\pi}{2}\le x\le\dfrac{3\pi}{4}$, then $\cos^{-1}\left(\dfrac{12}{13}\cos x+\dfrac{5}{13}\sin x\right)$ is equal to

(1) $x+\tan^{-1}\dfrac{5}{12}$ (2) $x-\tan^{-1}\dfrac{4}{3}$

(3) $x+\tan^{-1}\dfrac{4}{5}$ (4) $x-\tan^{-1}\dfrac{5}{12}$

3. Let $R=\{(1,2),(2,3),(3,3)\}$ be a relation defined on the set $\{1,2,3,4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

(1) 7 (2) 10 (3) 8 (4) 9

4. If the system of equations

$$(\lambda-1)x+(\lambda-4)y+\lambda z=5$$

$$\lambda x+(\lambda-1)y+(\lambda-4)z=7$$

$$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$$

has infinitely many solutions, then $\lambda^2+\lambda$ is equal to

(1) 20 (2) 10 (3) 6 (4) 12

5. The value of $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$ is

(1) 2/3 (2) 3/2 (3) 1 (4) 0

6. If the line $3x-2y+12=0$ intersects the parabola $4y=3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1) $\dfrac{\pi}{2}-\tan^{-1}\left(\dfrac{3}{2}\right)$ (2) $\tan^{-1}\left(\dfrac{9}{7}\right)$

(3) $\tan^{-1}\left(\dfrac{11}{9}\right)$ (4) $\tan^{-1}\left(\dfrac{4}{5}\right)$

7. The value of

$$\int_{e^2}^{e^4} \frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}}+e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx \text{ is}$$

(1) 1 (2) e^2 (3) $\log_e 2$ (4) (2)

8. Let the arc AC of a circle subtend a right angle at the centre O . If the point B on the arc AC,

divides the arc AC such that $\frac{\text{length of arcAB}}{\text{length of arc BC}}=\frac{1}{5}$, and $\overrightarrow{OC}=\alpha\overrightarrow{OA}+\beta\overrightarrow{OB}$, then

$\alpha+\sqrt{2}(\sqrt{3}-1)\beta$ is equal to

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$ (3) $5\sqrt{3}$ (4) $2\sqrt{3}$

9. If the function

$$f(x)=\begin{cases}\frac{2}{x}\{\sin(k_1+1)x+\sin(k_2-1)x\}, & x<0\\ 4, \quad x=0 & \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x>0\end{cases}$$

is continuous at $x=0$, then $k_1^2+k_2^2$ is equal to

(1) 5 (2) 10 (3) 20 (4) 8

10. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(1) –1200 (2) –1020 (3) –1080 (4) –120

11. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

(1) 35000 (2) 34000 (3) 36000 (4) 37000

12. Let a curve $y=f(x)$ pass through the points $(0,5)$ and $(\log_e 2,k)$. If the curve satisfies the differential equation $2(3+y)e^{2x}dx-\left(7+e^{2x}\right)dy=0$, then k is equal to

(1) 16 (2) 32 (3) 8 (4) (4)

13. Marks obtains by all the students of class 12 are presented in a freqency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18 , then the total number of students is

(1) 44 (2) 48 (3) 52 (4) 40

14. Let the position vectors of the vertices A, B and C of a tetrahedron A, B, C, D be $\hat{\mathbf{i}}+2\hat{j}+\hat{k}, \hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{\mathrm{i}}+\hat{\mathrm{j}}-\mathrm{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

(1) $\frac{1}{6}(12\hat{i}+12\hat{j}+\hat{k})$
(2) $\frac{1}{12}(7\hat{i}+4\hat{j}+3\hat{k})$
(3) $\frac{1}{6}(7\hat{\mathrm{i}}+12\hat{\mathrm{j}}+\mathrm{k})$
(4) $\frac{1}{2}(\hat{i}+4\hat{j}+7\hat{k})$

15. Let the area of a ΔPQR with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35 square units. If its orthocenter and centroid are $O\left(2,\frac{14}{5}\right)$ and $C(c,d)$ respectively, then $c+2d$ is equal to

(1) $\frac{7}{3}$
(2) $\frac{8}{3}$
(3) 2
(4) (3)

16. Let P be the foot of the perpendicular from the point $Q(10,-3,-1)$ on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$. Then the area of the right angled triangle P Q R, where R is the point (3,–2,1), is

(1) $8\sqrt{15}$
(2) $\sqrt{30}$
(3) $3\sqrt{30}$
(4) $9\sqrt{15}$

17. Let $\mathrm{I}(x)=\int\frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. If $\mathrm{I}(37)-\mathrm{I}(24)=\frac{1}{4}\left(\frac{1}{\mathrm{b}^{\frac{1}{13}}}-\frac{1}{\mathrm{c}^{\frac{1}{13}}}\right), \mathrm{b},\mathrm{c}\in\mathbb{N}$, then $3(\mathrm{b}+\mathrm{c})$ is equal to

(1) 40
(2) 39
(3) 26
(4) 22

18. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}, z\in C$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points $(0,0), \mathrm{C}$ and $(\alpha,0)$ is 11 square units, then α^2 equals:

(1) $\frac{121}{25}$
(2) 100
(3) $\frac{81}{25}$
(4) 50

19. If A,B, and $\left(\mathrm{adj}\left(\mathrm{A}^{-1}\right)+\mathrm{adj}\left(\mathrm{B}^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $\mathrm{A}\left(\mathrm{adj}\left(\mathrm{A}^{-1}\right)+\mathrm{adj}\left(\mathrm{B}^{-1}\right)\right)^{-1}\mathrm{B}$, is equal to

(1) $\frac{1}{|AB|}(\mathrm{adj}(B)+\mathrm{adj}(A))$
(2) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$
(3) $\mathrm{AB}^{-1}+\mathrm{A}^{-1}\mathrm{B}$
(4) $\mathrm{adj}\left(\mathrm{B}^{-1}\right)+\mathrm{adj}\left(\mathrm{A}^{-1}\right)$

20. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1 , two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5 , when both the dice are thrown together, is

(1) $\frac{2}{3}$ (2) $\frac{4}{9}$ (3) $\frac{1}{2}$ (4) $\frac{3}{5}$

SECTION-B

21. If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, where $a+c=15$ and $b=\frac{36}{5}$, a^2+c^2 is equal to

22. The sum of all rational terms in the expansion of $\left(1+2^{1/3}+3^{1/2}\right)^6$ is equal to ......

23. Let the circle C touch the line $x-y+1=0$, have the centre on the positive x -axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x+2y=1$. Let H be the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C . Then $2\alpha^2+3\beta^2$ is equal to ....

24. If the area of the larger portion bounded between the curves $x^2+y^2=25$ and $y=|x-1|$ is $\frac{1}{4}(b\pi+c), b,c\in\mathbb{N}$, then $b+c$ is equal to ....

25. If the set of all values of a, for which the equation $5x^3-15x-a=0$ has three distinct real roots, is the interval (α,β), then $\beta-2\alpha$ is equal to ....

NTA ANSWERS

1.	(1)	2.	(4)	3.	(1)	4.	(4)	5.	(3)	6.	(2)	7.	(1)
8.	(1)	9.	(2)	10.	(3)	11.	(3)	12.	(3)	13.	(1)	14.	(3)
15.	(4)	16.	(3)	17.	(2)	18.	(2)	19.	(1)	20.	(3)	21.	117
22.	612	23.	16	24.	77	25.	30						