JEE-MAIN EXAM JANUARY, 2025

Date: - 24-01-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Let circle C be the image of $x^2+y^2-2x+4y-4-0$ in the line $2x-3y+5=0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha,\beta)$, with $\beta<4$, lies on C such that the length of the arc AB is $(1/6)^{\text{th}}$ of the perimeter of C, then $\beta-\sqrt{3}\alpha$ is equal to

(1) $4-\sqrt{3}$ (2) 3 (3) 4 (4) $3+\sqrt{3}$

2. The area of the region $\{(x,y):x^2+4x+2\le y\le|x+2|\}$ is equal to

(1) 24/5 (2) 5 (3) 20/3 (4) 7

3. Consider the region $R=\left\{(x,y):x\le y\le 9-\frac{11}{3}x^2, x\ge 0\right\}$. The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is:

(1) $\frac{567}{121}$ (2) $\frac{821}{123}$ (3) $\frac{730}{119}$ (4) $\frac{625}{111}$

4. Let $S_n=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term -p and common difference p is $\sqrt{2026S_{2025}}$, then the absolute difference between 20^{th} and 15^{th} terms of the A.P. is

(1) 45 (2) 20 (3) 25 (4) 90

5. Let the line passing through the points (-1, 2, 1) and parallel to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ intersect the line $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}$ at the point P. Then the distance of P from the point Q(4, -5, 1) is

(1) $5\sqrt{6}$ (2) $5\sqrt{5}$ (3) 5 (4) 10

6. Let $\vec{\mathbf{a}}=\hat{i}+2\hat{j}+3\hat{k}, \vec{\mathbf{b}}=3\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a}\cdot\vec{c}=5$, then $|\vec{c}|$ is equal to

(1) 18 (2) 16 (3) $\sqrt{\frac{11}{6}}$ (4) $\frac{1}{3\sqrt{2}}$

7. If the system of equations

$$2x-y+z=4$$

$$5x+\lambda y+3z=12$$

$$100x-47y+\mu z=212$$

has infinitely many solutions, then $\mu-2\lambda$ is equal to

(1) 55 (2) 57 (3) 59 (4) 56

8. If α and β are the roots of the equation $2z^2-3z-2i=0$, where $i=\sqrt{-1}$, then $16\cdot\mathrm{Re}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)\cdot\mathrm{Im}\left(\frac{\alpha^{19}+\beta^{19}+\alpha^{11}+\beta^{11}}{\alpha^{15}+\beta^{15}}\right)$ is equal to

(1) 398 (2) 441 (3) 312 (4) 409

9. Let y = y(x) be the solution of the differential equation $\left(xy-5x^2\sqrt{1+x^2}\right)dx+\left(1+x^2\right)dy=0, y(0)=0$.

Then $y(\sqrt{3})$ is equal to Options

(1) $\frac{5\sqrt{3}}{2}$ (2) $\sqrt{\frac{14}{3}}$ (3) $\sqrt{\frac{15}{2}}$ (4) $2\sqrt{2}$

10. Let $f(x)=\frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+\ldots+f\left(\frac{59}{15}\right)\right)$ is equal to

(1) 102 (2) 92 (3) 118 (4) 108

11. For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2=371$.

He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is

(1) 5 (2) 7 (3) 9 (4) 4

12. Let the product of the focal distances of the point $\left(\sqrt{3},\frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$, be $\frac{7}{4}$.

Then the absolute difference of the eccentricities of two such ellipses is

(1) $\frac{1-\sqrt{3}}{\sqrt{2}}$ (2) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$ (3) $\frac{1-2\sqrt{2}}{\sqrt{3}}$ (4) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

13. Let $f:\mathbb{R}-\{0\}\to\mathbb{R}$ be a function such that $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}$.

If the $\lim_{x\to 0}\left(\frac{1}{\alpha x}+f(x)\right)=\beta;\alpha,\beta\in\mathbb{R}$, then $\alpha+2\beta$ is equal to

(1) 6 (2) 5 (3) 4 (4) 3

14. Let the lines $3x-4y-\alpha=0, 8x-11y-33=0$, and $2x-3y+\lambda=0$ be concurrent. If the image of the point $(1,2)$ in the line $2x-3y+\lambda=0$ is $\left(\frac{57}{13},\frac{-40}{13}\right)$, then $|\alpha\lambda|$ is equal to

(1) 101 (2) 91 (3) 113 (4) 84

15. Let in a ΔABC, the length of the side AC be 6, the vertex B be (1, 2, 3) and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of ΔABC is:

(1) 17 (2) 21 (3) 56 (4) 42

16. The product of all the rational roots of the equation $\left(x^2-9x+11\right)^2-(x-4)(x-5)=3$, is equal to

(1) 21 (2) 28 (3) 14 (4) 7

17. For some $\mathrm{n}\neq 10$, let the coefficients of the 5th, 6th and 7th terms in the binomial expansion of $(1+x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1+x)^{n+4}$ is:

(1) 70 (2) 10 (3) 20 (4) 35

18. $\lim\limits_{x\to 0}\operatorname{cosec} x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$ is:

(1) $\frac{1}{2\sqrt{5}}$ (2) 0 (3) $-\frac{1}{2\sqrt{5}}$ (4) $\frac{1}{\sqrt{15}}$

19. A and B alternately throw a pair of dice. A wins if he throws a sum of 5 before B throws a sum of 8, and B wins if he throws a sum of 8 before A throws a sum of 5. The probability, that A wins if A makes the first throw, is

(1) $\frac{8}{19}$ (2) $\frac{9}{19}$ (3) $\frac{8}{17}$ (4) $\frac{9}{17}$

20. If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, m,n>0$, then $I(9,14)+I(10,13)$ is

(1) I(1, 13) (2) I(9, 1) (3) I(19, 27) (4) I(9, 13)

SECTION-B

21. Let A be a 3×3 matrix such that $X^TAX=O$ for all nonzero 3×1 matrices $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$. If

$\mathrm{A}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\4\\-5\end{bmatrix}, \mathrm{A}\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}0\\4\\-8\end{bmatrix}$, and $\det(\operatorname{adj}(2(\mathrm{A}+\mathrm{I})))=2^\alpha 3^\beta 5^\gamma, \alpha,\beta,\gamma\in\mathbb{N}$, then $\alpha^2+\beta^2+\gamma^2$ is

22. The number of 3-digit numbers, that are divisible by 2 and 3, but not divisible by 4 and 9, is

23. If for some $\alpha, \beta; \alpha \leq \beta, \alpha + \beta = 8$ and $\sec^2\left(\tan^{-1}\alpha\right) + \operatorname{cosec}^2\left(\cot^{-1}\beta\right) = 36$, then $\alpha^2 + \beta$ is ________.

24. Let $S = \{p_1, p_2 \ldots, p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs (x, y), $x \in S$, $y \in A$, such that x divides y, is________.

25. Let f be a differentiable function such that $2(x+2)^2 f(x) - 3(x+2)^2 = 10\int_0^x (t+2)f(t)dt, x \geq 0$.

Then $f(2)$ is equal to ________.

NTA ANSWERS

1.	**(3)**	**2.**	**(3)**	**3.**	**(1)**	**4.**	**(3)**	**5.**	**(2)**	**6.**	**(3)**	**7.**	**(2)**
8.	**(2)**	**9.**	**(1)**	**10.**	**(3)**	**11.**	**(2)**	**12.**	**(4)**	**13.**	**(3)**	**14.**	**(2)**
15.	**(2)**	**16.**	**(3)**	**17.**	**(4)**	**18.**	**(3)**	**19.**	**(2)**	**20.**	**(4)**	**21.**	**(44)**
22.	**(125)**	**23.**	**(14)**	**24.**	**(5120)**	**25.**	**(19)**						