JEE-MAIN EXAM JANUARY, 2025

Date: - 24-01-2025 (SHIFT-2)

MATHEMATICS

SECTION-A

1. Let [x] denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x)-[x]+|x-2|,-2<x<3$, is not continuous and not differentiable. Then $m+n$ is equal to :

(1) 6 (2) 7 (3) 8 (4) 9

2. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2A=5B$, then n is equal to :

(1) 19 (2) 21 (3) 20 (4) 22

3. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is $(3,1)$ is :

(1) $4x+122y=134$ (2) $5x+16y=31$

(3) $48x+25y=169$ (4) $25x+101y=176$

4. If the system of equations

$x+2y-3z=2$

$2x+\lambda y+5z=5$

$14x+3y+\mu z=33$

has infinitely many solutions, then $\lambda+\mu$ is equal to :

(1) 10 (2) 12 (3) 13 (4) 11

5. The area of the region enclosed by the curves $y=e^x, y=|e^x-1|$ and y-axis is :

(1) $1-\log_e 2$ (2) $\log_e 2$ (3) $2\log_e 2-1$ (4) $1+\log_e 2$

6. If $7=5+\frac{1}{7}(5+\alpha)+\frac{1}{7^2}(5+2\alpha)+\frac{1}{7^3}(5+3\alpha)+\ldots\ldots\ldots\ldots$, then the value of α is :

(1) $\frac{6}{7}$ (2) 1 (3) $\frac{1}{7}$ (4) 6

7. The number of real solution(s) of the equation $x^2+3x+2=\min\{|x-3|,|x+2|\}$ is :

(1) 0 (2) 3 (3) 1 (4) 2

8. For some a, b, let $f(x)=\begin{vmatrix} a+\frac{\sin x}{x} & 1 & \mathbf{b} \\ \mathbf{a} & 1+\frac{\sin x}{x} & \mathbf{b} \\ \mathbf{a} & 1 & \mathbf{b}+\frac{\sin x}{x} \end{vmatrix}, x \neq 0, \lim\limits_{x\to 0} f(x)=\lambda+\mu\mathbf{a}+\nu\mathbf{b}$. Then $(\lambda+\mu+\nu)^2$ is equal to :

(1) 9 (2) 36 (3) 16 (4) 25

9. Let $A=\left[a_{ij}\right]$ be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability $P(E)$ is :

(1) $\frac{5}{8}$ (2) $\frac{3}{16}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

10. Let $A=\left\{x\in(0,\pi)-\left\{\frac{\pi}{2}\right\}:\log_{(2/\pi)}|\sin x|+\log_{(2/\pi)}|\cos x|=2\right\}$ and $B=\left\{x\geq 0:\sqrt{x}\left(\sqrt{x}-4\right)-3|\sqrt{x}-2|+6=0\right\}$. Then $n(A\cup B)$ is equal to :

(1) 4 (2) 2 (3) 6 (4) 8

11. Let $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}, \vec{b}=\vec{a}\times\left(\hat{i}-2\hat{k}\right)$ and $\vec{c}=\vec{b}\times\hat{k}$. Then the projection of $\vec{c}-2\hat{j}$ on $\vec{a}$ is:

(1) $2\sqrt{7}$ (2) $\sqrt{14}$ (3) $3\sqrt{7}$ (4) $2\sqrt{14}$

12. The function $f:(-\infty,\infty)\to(-\infty,1)$, defined by $f(x)=\frac{2^x-2^{-x}}{2^x+2^{-x}}$ is :

(1) Both one-one and onto (2) Onto but not one-one
(3) Neither one-one nor onto (4) One-one but not onto

13. If the equation of the parabola with vertex $V\left(\frac{3}{2},3\right)$ and the directrix $x+2y=0$ is $\alpha x^2+\beta y^2-\gamma xy-30x-60y+225=0$, then $a+\beta+\gamma$ is equal to :

(1) 9 (2) 7 (3) 6 (4) 8

14. Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $a\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to :

(1) 3 (2) 6 (3) 1 (4) 4

15. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to :

(1) 9100 (2) 8925 (3) 8750 (4) 8575

16. Let (2,3) be the largest open interval in which the function $f(x)=2\log_e(x-2)-x^2+ax+1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x)=(x-1)^3(x+2-a)^2$ is strictly decreasing. Then 100(a + b - c) is equal to :

(1) 420 (2) 360 (3) 160 (4) 280

17. If $\alpha>\beta>\gamma>0$, then the expression

$$\cot^{-1}\left\{\beta+\frac{(1+\beta^2)}{(\alpha-\beta)}\right\}+\cot^{-1}\left\{\gamma+\frac{(1+\gamma^2)}{(\beta-\gamma)}\right\}+\cot^{-1}\left\{\alpha+\frac{(1+\alpha^2)}{(\gamma-\alpha)}\right\}$$ is equal to :

(1) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$ (2) 0 (3) π (4) 3π

18. In an arithmetic progression, if $S_{40}=1030$ and $S_{12}=57$, then $S_{30}-S_{10}$ is equal to :

(1) 505 (2) 515 (3) 510 (4) 525

19. Let the points $\left(\frac{11}{2},\alpha\right)$ lie on or inside the triangle with sides $x+y=11, x+2y=16$ and $2x+3y=29$. Then the product of the smallest and the largest values of α is equal to :

(1) 22 (2) 55 (3) 44 (4) 33

20. Let $f:(0,\infty)\to\mathbf{R}$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2f'(x)=2xf(x)+3$, with $f(1)=4$. Then $2f(2)$ is equal to :

(1) 19 (2) 39 (3) 29 (4) 23

SECTION-B

21. Let P be the image of the point $Q(7,-2,5)$ in the line $L:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ and $R(5,p,q)$ be a point on L. Then the square of the area of ΔPQR is _____.

22. If $\int\frac{2x^2+5x+9}{\sqrt{x^2+x+1}}dx=x\sqrt{x^2+x+1}+\alpha\sqrt{x^2+x+1}+\beta\log_e\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$, where C is the constant of integration, then $\alpha+2\beta$ is equal to _____ .

23. Number of functions $f:\{1,2,\ldots,100\}\to\{0,1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to _____ .

24. Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : -\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their eccentricities be $e_1 = \sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to _____.

25. Let $y = y(x)$ be the solution of the differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y \sin x, x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{3}\right) = 0$, then $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to _____ .

NTA ANSWERS

1.	(3)	2.	(2)	3.	(3)	4.	(2)	5.	(1)	6.	(4)	7.	(4)
8.	(3)	9.	(4)	10.	(4)	11.	(4)	12.	(4)	13.	(1)	14.	(1)
15.	(2)	16.	(2)	17.	(3)	18.	(2)	19.	(4)	20.	(2)	21.	(957)
22.	(16)	23.	(392)	24.	(55)	25.	(1)						