JEE-MAIN EXAM JANUARY, 2025

Date: - 28-01-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Two number k_1 and k_2 are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}, (i = \sqrt{-1})$ is non-zero, equals

(1) $\frac{1}{4}$ (2) $\frac{3}{4}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

2. If the image of the point $(4,4,3)$ in the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 9 (2) 8 (3) 12 (4) 7

3. The sum of all local minimum values of the function

$$f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7+2|x|), & -1 \le x \le 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$$ is

(1) $\frac{131}{72}$ (2) $\frac{167}{72}$ (3) $\frac{171}{72}$ (4) $\frac{157}{72}$

4. Three defective oranges are accidently mixed with seven good ones and on looking at them, it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denote the number of defective oranges, then the variance of x is

(1) $28/75$ (2) $14/25$ (3) $18/25$ (4) $26/75$

5. Let for some function $y = f(x), \int_0^x t f(t)dt = x^2 f(x), x > 0$ and $f(2) = 3$. Then $f(6)$ is equal to

(1) 1 (2) 2 (3) 3 (4) 6

6. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0 , 1,2,3,4,5,6,7, such that the sum of their first and last digits should not be more than 8 , is

(1) 4608 (2) 5719 (3) 4607 (4) 5720

7. Let $\langle a_n \rangle$ be a sequence such that $a_0 = 0, a_1 = \frac{1}{2}$ and $2a_{n+2} = 5a_{n+1} - 3a_n, n = 0,1,2,3,\ldots$ Then $\sum_{k=1}^{100} a_k$ is equal to

(1) $3a_{100} + 100$ (2) $3a_{99} - 100$ (3) $3a_{99} + 100$ (4) $3a_{100} - 100$

8. Let $A(x, y, z)$ be a point in xy-plane, which is equidistant from three points (0,3,2),(2,0,3) and (0,0,1). Let $B = (1, 4, -1)$ and $C = (2, 0, -2)$. Then among the statements

(S1): ΔABC is an isosceles right angled triangle, and

(S2) : the area of ΔABC is $\frac{9\sqrt{2}}{2}$

(1) only (S2) is true (2) both are false (3) both are true (4) only (S1) is true

9. The relation $R = \{(x, y) : x, y \in \mathbb{Z}$ and $x + y$ is even $\}$ is:

(1) an equivalence relation

(2) symmetric and transitive but not reflexive

(3) reflexive and symmetric but not transitive

(4) reflexive and transitive but not symmetric

10. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$ is equal to:

(1) 1 (2) $\frac{33}{65}$ (3) $\frac{32}{65}$ (4) 0

11. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$f(x) = (2 + 3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If

$f(x + y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28\sum_{i=1}^{5}|f(i)|$ is

(1) 545 (2) 715 (3) 735 (4) 675

12. Let ${}^{n}C_{r-1} = 28, {}^{n}C_{r} = 56$ and ${}^{n}C_{r+1} = 70$. Let $A(4\cos t, 4\sin t), B(2\sin t, -2\cos t)$ and $C\left(3r - n, r^2 - n - 1\right)$ be the vertices of a triangle ABC , where t is a parameter. If $(3x - 1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC , then α equals

(1) 18 (2) 8 (3) 6 (4) 20

13. The area (in sq. units) of the region $\left\{(x, y) : 0 \leq y \leq 2|x| + 1, 0 \leq y \leq x^2 + 1, |x| \leq 3\right\}$ is

(1) $\frac{32}{3}$ (2) $\frac{17}{3}$

(3) $\frac{64}{3}$ (4) $\frac{80}{3}$

14. Let the equation of the circle, which touches x-axis at the point $(a, 0), a > 0$ and cuts off an intercept of length b on $y-axis$ be $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$. If the circle lies below $x-axis$, then the ordered pair $\left(2a, b^2\right)$ is equal to

(1) $\left(\alpha, \beta^2 + 4\gamma\right)$ (2) $\left(\gamma, \beta^2 - 4\alpha\right)$ (3) $\left(\alpha, \beta^2 - 4\gamma\right)$ (4) $\left(\gamma, \beta^2 + 4\alpha\right)$

15. Let O be the origin, the point A be $z_1 = \sqrt{3} + 2\sqrt{2}i$, the point $B(z_2)$ be such that $\sqrt{3}|z_2| = |z_1|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$. Then

(1) ABO is a scalene triangle
(2) area of triangle ABO is $\frac{11}{4}$
(3) area of triangle ABO is $\frac{11}{\sqrt{3}}$
(4) ABO is an obtuse angled isosceles triangle

16. If $f(x) = \frac{2^x}{2^x + \sqrt{2}}, x \in \mathbb{R}$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to

(1) 82
(2) $\frac{81}{2}$
(3) $81\sqrt{2}$
(4) 41

17. Let ABCD be a trapezium whose vertices lie on the parabola $y^2 = 4x$. Let the sides AD and BC of the trapezium be parallel to y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point $(1,0)$, then the area of ABCD is

(1) $\frac{75}{4}$
(2) $\frac{125}{8}$
(3) $\frac{25}{2}$
(4) $\frac{75}{8}$

18. The sum, of the squares of all the roots of the equation $x^2 + |2x-3| - 4 = 0$, is

(1) $3(3-\sqrt{2})$
(2) $3(2-\sqrt{2})$
(3) $6(2-\sqrt{2})$
(4) $6(3-\sqrt{2})$

19. Let T_r be the r^{th} term of an A.P. If for some $m, T_m = \frac{1}{25}, T_{25} = \frac{1}{20}$, and $20\sum_{r=1}^{25} T_r = 13$, then $5m\sum_{r=m}^{2m} T_r$ is equal to

(1) 112
(2) 142
(3) 98
(4) 126

20. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96x^2\cos^2 x}{(1+e^x)} dx = \pi(\alpha\pi^2 + \beta), \alpha, \beta \in \mathbb{Z}$, then $(\alpha+\beta)^2$ equals

(1) 196
(2) 100
(3) 144
(4) 64

SECTION-B

21. Let $f(x) = \begin{cases} 3x, & x < 0 \\ \min\{1 + x + [x], x + 2[x]\}, & 0 \le x \le 2 \\ 5, & x > 2 \end{cases}$

where [.] denotes greatest integer function. If α and β are the number of points, where f is not continuous and is not differentiable, respectively, then $\alpha + \beta$ equals ___ .

22. Let $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ be an ellipse. Ellipses E_i 's are constructed such that their centres and eccentricities are same as that of E_1, and the length of minor axis of E_i is the length of major axis of $E_{i+1} (i \geq 1)$. If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty} A_i\right)$, is equal to ______ .

23. Let M denote the set of all real matrices of order 3×3 and let $S = \{-3, -2, -1, 1, 2\}$. Let

$S_1 = \left\{A = \left[a_{ij}\right] \in M : A = A^T \text{ and } a_{ij} \in S, \forall i, j\right\}$,

$S_2 = \left\{A = \left[a_{ij}\right] \in M : A = -A^T \text{ and } a_{ij} \in S, \forall i, j\right\}$,

$S_3 = \left\{A = \left[a_{ij}\right] \in M : a_{11} + a_{22} + a_{33} = 0 \text{ and } a_{ij} \in S, \forall i, j\right\}$.

If $n\left(S_1 \cup S_2 \cup S_3\right) = 125\alpha$, then α equals ______.

24. Let $\vec{a} = \hat{i} + \hat{j} + k, \vec{b} = 2\hat{i} + 2\hat{j} + k$ and $\vec{d} = \vec{a} \times \vec{b}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - 2\vec{a}|^2 = 8$ and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $|10 - 3\vec{b} \cdot \vec{c}| + |\vec{d} \times \vec{c}|^2$ is equal to______.

25. If $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1} \quad {}^{12}C_{2r-1}$, then the distance of the point $(12, \sqrt{3})$ from the line $\alpha x - \sqrt{3}y + 1 = 0$ is ______.

NTA ANSWERS

1.	**(2)**	**2.**	**(1)**	**3.**	**(4)**	**4.**	**(1)**	**5.**	**(1)**	**6.**	**(3)**	**7.**	**(4)**
8.	**(4)**	**9.**	**(1)**	**10.**	**(4)**	**11.**	**(4)**	**12.**	**(4)**	**13.**	**(3)**	**14.**	**(3)**
15.	**(4)**	**16.**	**(2)**	**17.**	**(1)**	**18.**	**(3)**	**19.**	**(4)**	**20.**	**(2)**	**21.**	**(5)**
22.	**(54)**	**23.**	**(1613)**	**24.**	**(6)**	**25.**	**(5)**						