JEE-MAIN EXAM JANUARY, 2025

Date: - 28-01-2025 (SHIFT-2)

MATHEMATICS

SECTION-A

1. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S , one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is :

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{3}$

2. If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of ΔPAB lies on the line :

(1) $4x-9y=12$ (2) $6x-9y=20$ (3) $9x-9y=32$ (4) $x+9y=36$

3. For positive integers n, if $4a_n=\left(n^2+5n+6\right)$ and $S_n=\sum_{k=1}^{n}\left(\frac{1}{a_k}\right)$, then the value of $507S_{2025}$ is:

(1) 540 (2) 135 (3) 675 (4) 1350

4. Two equal sides of an isosceles triangle are along $-x+2y=4$ and $x+y=4$. If m is the slope of its third side, then the sum, of all possible distinct values of m, is :

(1) $-2\sqrt{10}$ (2) 6 (3) 12 (4) -6

5. The square of the distance of the point $\left(\frac{15}{7},\frac{32}{7},7\right)$ from the line $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ in the direction of the vector $\hat{i}+4\hat{j}+7\hat{k}$ is:

(1) 41 (2) 66 (3) 44 (4) 54

6. Let $f:[0,3]\to$ A be defined by $f(x)=2x^3-15x^2+36x+7$ and $g:[0,\infty)\to B$ be defined by $g(x)=\frac{x^{2025}}{x^{2025}+1}$. If both the functions are onto and $S=\{x\in Z: x\in A \text{ or } x\in B\}$, then $n(S)$ is equal to:

(1) 29 (2) 36 (3) 30 (4) 31

7. Let f be a real valued continuous function defined on the positive real axis such that $g(x)=\int_0^x tf(t)dt$ If $g\left(x^3\right)=x^6+x^7$, then value of $\sum_{r=1}^{15} f\left(r^3\right)$ is :

(1) 340 (2) 310 (3) 320 (4) 270

8. Let $\mathrm{A, B, C}$ be three points in $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\mathrm{a}\hat{i}+(1-\mathrm{a})\hat{j}$ respectively with respect to the origin O. If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is :

(1) 9/2 (2) 1 (3) 2 (4) 0

9. Let $f:\mathbf{R}\to\mathbf{R}$ be a twice differentiable function such that $f(2)=1$. If $\mathrm{F}(x)=xf(x)$ for all $x\in\mathbf{R}$, $\int_0^2 x\mathrm{F}'(x)\mathrm{d}x=6$ and $\int_0^2 x^2\mathrm{F}''(x)\mathrm{d}x=40$, then $\mathrm{F}'(2)+\int_0^2\mathrm{F}(x)\mathrm{d}x$ is equal to :

(1) 9 (2) 11 (3) 15 (4) 13

10. Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $(\sqrt[4]{3}+\sqrt[3]{4})^{12}$. Then $p+q$ is equal to :

(1) 299 (2) 287 (3) 283 (4) 295

11. If the components of $\vec{\mathbf{a}}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ along and perpendicular to $\vec{\mathbf{b}}=3\hat{i}+\hat{j}-\hat{k}$ respectively, are $\frac{16}{11}(3\hat{i}+\hat{j}-\hat{k})$ and $\frac{1}{11}(-4\hat{i}-5\hat{j}-17\hat{k})$, then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 23 (2) 26 (3) 16 (4) 18

12. Let $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ and $\mathrm{P}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \theta>0$. If $\mathrm{B}=\mathrm{PAP}^{\mathrm{T}}, \mathrm{C}=\mathrm{P}^{\mathrm{T}}\mathrm{B}^{10}\mathrm{P}$ and the sum of the diagonal elements of C is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is :

(1) 65 (2) 127 (3) 2049 (4) 258

13. If $\alpha+i\beta$ and $\gamma+i\delta$ are the roots of $x^2-(3-2i)x-(2i-2)=0, i=\sqrt{-1}$, then $\alpha\gamma+\beta\delta$ is equal to :

(1) –2 (2) 2 (3) –6 (4) 6

14. If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\}=a\sqrt{3}+b, a, b\in Z$, then a^2+b^2 is equal to :

(1) 10 (2) 2 (3) 4 (4) 8

15. The area of the region bounded by the curves $x\left(1+y^2\right)=1$ and $y^2=2x$ is:

(1) $\frac{1}{2}\left(\frac{\pi}{2}-\frac{1}{3}\right)$ (2) $\frac{\pi}{2}-\frac{1}{3}$ (3) $2\left(\frac{\pi}{2}-\frac{1}{3}\right)$ (4) $\frac{\pi}{4}-\frac{1}{3}$

16. Let $f:\mathbf{R}-\{0\}\to(-\infty,1)$ be a polynomial of degree 2, satisfying $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$. If $f(\mathrm{K})=-2\mathrm{K}$, then the sum of squares of all possible values of K is :

(1) 6 (2) 9 (3) 7 (4) 1

17. If $f(x)=\int\frac{1}{x^{1/4}\left(1+x^{1/4}\right)}\mathrm{d}x, f(0)=-6$, then $f(1)$ is equal to :

(1) $4\left(\log_e 2+2\right)$ (2) $2-\log \mathrm{e}^2$ (3) $\log_e 2+2$ (4) $4\left(\log_{e^2} 2-2\right)$

18. If the midpoint of a chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $(\sqrt{2},4/3)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is :

(1) 20 (2) 18 (3) 26 (4) 22

19. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability, that the ball is drawn from Bag B_2, is

(1) $\frac{1}{3}$ (2) $\frac{2}{5}$ (3) $\frac{2}{3}$ (4) $\frac{4}{15}$

20. Let [x] denote the greatest integer less than or equal to x. Then the domain of $f(x)=\sec^{-1}(2[x]+1)$ is :

(1) $(-\infty,\infty)$ (2) $(-\infty,-1]\cup[1,\infty)$

(3) $(-\infty,-1]\cup[0,\infty)$ (4) $(-\infty,\infty)-\{0\}$

SECTION-B

21. Let $f(x)=\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\left(\frac{\tan\left(x/2^{r+1}\right)+\tan^3\left(x/2^{r+1}\right)}{1-\tan^2\left(x/2^{r+1}\right)}\right)$. Then $\lim\limits_{x\to 0}\frac{\mathrm{e}^x-\mathrm{e}^{f(x)}}{(x-f(x))}$ is equal to ________.

22. Let A and B be the two points of intersection of the line $y+5=0$ and the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If d denotes the distance between A and B, and a denotes the area of $\Delta\mathrm{SAB}$, where S is the focus of the parabola $y^2=4x$, then the value of $(a+d)$ is ______ .

23. The interior angles of a polygon with n sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to ______.

24. If $y = y(x)$ is the solution of the differential equation,

$$\sqrt{4-x^2}\frac{dy}{dx} = \left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2 - y\right)\sin^{-1}\left(\frac{x}{2}\right), -2 \le x \le 2, y(2) = \frac{\pi^2-8}{4}$$, then $y^2(0)$ is equal to

25. The number of natural numbers, between 212 and 999, such that the sum of their digits is 15, is ________.

NTA ANSWERS

1.	**(2)**	**2.**	**(2)**	**3.**	**(3)**	**4.**	**(2)**	**5.**	**(2)**	**6.**	**(3)**	**7.**	**(2)**
8.	**(2)**	**9.**	**(2)**	**10.**	**(3)**	**11.**	**(2)**	**12.**	**(1)**	**13.**	**(2)**	**14.**	**(4)**
15.	**(2)**	**16.**	**(1)**	**17.**	**(4)**	**18.**	**(4)**	**19.**	**(4)**	**20.**	**(1)**	**21.**	**(1)**
22.	**(14)**	**23.**	**(20)**	**24.**	**(4)**	**25.**	**(64)**						