JEE-MAIN EXAM JANUARY, 2025

Date: - 29-01-2025 (SHIFT-1)

MATHEMATICS

SECTION-A

1. Let the area of the region $\{(x,y): 2y \le x^2+3, y+|x| \le 3, y \ge |x-1|\}$ be A . Then 6 A is equal to :

(1) 14 (2) 16 3. 18 (4) 12

2. Let P be the set of seven digit numbers with sum of their digits equal to 11 . If the numbers in P are formed by using the digits 1,2 and 3 only, then the number of elements in the set P is :

(1) 161 (2) 164 (3) 173 (4) 158

3. The least value of n for which the number of integral terms in the Binomial expansion of $(\sqrt[3]{7}+\sqrt[12]{11})^n$ is 183 , is :

(1) 2172 (2) 2184 (3) 2196 (4) 2148

4. Define a relation R on the interval $\left[0,\frac{\pi}{2}\right)$ by $x\text{R}y$ if and only if $\sec^2 x-\tan^2 y=1$. Then R is :

(1) both reflexive and transitive but not symmetric

(2) both reflexive and symmetric but not transitive

(3) an equivalence relation

(4) reflexive but neither symmetric not transitive

5. Let the ellipse $\text{E}_1:\frac{x^2}{\text{a}^2}+\frac{y^2}{\text{b}^2}=1, \text{a}>\text{b}$ and $\text{E}_2:\frac{x^2}{\text{A}^2}+\frac{y^2}{\text{B}^2}=1, \text{A}<\text{B}$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A,B,C and D, then the area of the quadrilateral ABCD equals :

(1) $\frac{12\sqrt{6}}{5}$ (2) $6\sqrt{6}$

(3) $\frac{24\sqrt{6}}{5}$ (4) $\frac{18\sqrt{6}}{5}$

6. Let $y = y(x)$ be the solution of the differential equation

$\cos x\left(\log_e(\cos x)\right)^2 dy + \left(\sin x - 3y\sin x\log_e(\cos x)\right)dx = 0, x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right) = \frac{-1}{\log_e 2}$, then

$y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{1}{\log_e(4) - \log_e(3)}$ (2) $\frac{1}{\log_e(3) - \log_e(4)}$

(3) $\frac{2}{\log_e(3) - \log_e(4)}$ (4) $-\frac{1}{\log_e(4)}$

7. The number of solutions of the equation $\left(\frac{9}{x} - \frac{9}{\sqrt{x}} + 2\right)\left(\frac{2}{x} - \frac{7}{\sqrt{x}} + 3\right) = 0$ is :

(1) 4 (2) 3 (3) 2 (4) 1

8. The integral $80\int_0^{\frac{\pi}{4}}\left(\frac{\sin\theta + \cos\theta}{9 + 16\sin 2\theta}\right)d\theta$ is equal to :

(1) $2\log_e 3$ (2) $3\log_e 4$ (3) $6\log_e 4$ (4) $4\log_e 3$

9. Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B . If the line perpendicular to AB and passing through the mid point of the chord AB intersects the circle at C and D, then the area of the quadrilateral ADBC is equal to:

(1) $2\sqrt{14}$ (2) $5\sqrt{7}$ (3) $\sqrt{14}$ (4) $3\sqrt{7}$

10. Consider an A. P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800 . Then its 11^{th} term is :

(1) 122 (2) 90 (3) 84 (4) 108

11. The value of $\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}\right)$ is:

(1) $4/3$ (2) 2 (3) $7/3$ (4) $5/3$

12. Let M and m respectively be the maximum and the minimum values of

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in \mathrm{R}$$

Then $M^4 - m^4$ is equal to :

(1) 1040 (2) 1295 (3) 1280 (4) 1215

13. Let $|z_1-8-2i|\le 1$ and $|z_2-2+6i|\le 2, z_1, z_2 \in \mathbf{C}$. Then the minimum value of $|z_1-z_2|$ is:

(1) 3 (2) 10 (3) 7 (4) 13

14. Two parabolas have the same focus $(4,3)$ and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to :

(1) 192 (2) 384 (3) 392 (4) 96

15. Let ABC be a triangle formed by the lines $7x-6y+3=0, x+2y-31=0$ and $9x-2y-19=0$. Let the point (h, k) be the image of the centroid of ΔABC in the line $3x+6y-53=0$. Then h^2+k^2+hk is equal to :

(1) 40 (2) 47 (3) 37 (4) 36

16. Let $x_1, x_2, \ldots, x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i-2)=30, \sum_{i=1}^{10}(x_i-\beta)^2=98, \beta>2$, and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1-1)+4\beta$, $2(x_2-1)+4\beta, \ldots, 2(x_{10}-1)+4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

(1) 110 (2) 90 (3) 120 (4) 100

17. Let $A=[a_{ij}]=\begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$.

If A_{ij} is the cofactor of $a_{ij}, C_{ij}=\sum_{k=1}^{2} a_{ik}A_{jk}, 1\le i, j\le 2$, and $C=[C_{ij}]$, then 8|C| is equal to :

(1) 222 (2) 242 (3) 288 (4) 262

18. Let $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}, \vec{b}=3\hat{i}-5\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$ and $(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c})=168$. Then the maximum value of $|\vec{c}|^2$ is :

(1) 462 (2) 77 (3) 308 (4) 154

19. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{\mathbf{b}}=2\hat{i}+7\hat{j}+3\hat{k}$. Let $L_1: \vec{r}=(-\hat{i}+2\hat{j}+\hat{k})+\lambda\vec{\mathbf{a}}, \lambda\in\mathbf{R}$ and $L_2: \vec{r}=(\hat{j}+\hat{k})+\mu\vec{b}, \mu\in\mathbf{R}$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2, and is parallel to $\vec{a}+\vec{b}$, then L_3 passes through the point :

(1) (8,26,12) (2) (5,17,4) (3) (2,8,5) (4) (-1,-1,1)

20. Let $L_1: \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{2}$ and $L_2: \frac{x+1}{-1}=\frac{y-2}{2}=\frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α,β,γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha-11\beta-8\gamma|$ equals :

(1) 16 (2) 20 (3) 25 (4) 18

SECTION-B

21. Let [t] be the greatest integer less than or equal to t. Then the least value of $p \in \mathbb{N}$ for which

$$\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$$ is equal to ________.

22. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is _________.

23. Let $f:(0,\infty)\to \mathbf{R}$ be a twice differentiable function. If for some $a \neq 0, \int_0^1 f(\lambda x)d\lambda = af(x), f(1)=1$ and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to _________ .

24. Let $S=\{x:\cos^{-1}x=\pi+\sin^{-1}x+\sin^{-1}(2x+1)\}$. Then $\sum_{x\in S}(2x-1)^2$ is equal to ______.

25. Let $S=\{m\in \mathbf{Z}: A^{m^2}+A^m=3I-A^{-6}\}$, where $A=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$. Then n(S) is equal to ______.

NTA ANSWERS

1.	(1)	2.	(1)	3.	(2)	4.	(3)	5.	(3)	6.	(2)	7.	(1)
8.	(4)	9.	(1)	10.	(2)	11.	(4)	12.	(3)	13.	(3)	14.	(1)
15.	(3)	16.	(4)	17.	(2)	18.	(3)	19.	(1)	20.	(3)	21.	(24)
22.	(1405)	23.	(112)	24.	(5)	25.	(2)						