JEE-MAIN EXAM JANUARY, 2025

Date: - 29-01-2025 (SHIFT-2)

MATHEMATICS

SECTION-A

1. Let the line $x+y=1$ meet the axes of x and y at A and B, respectively. A right angled triangle A M N is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $\mathrm{AN}: NB=\lambda:1$, then the sum of all possible value(s) of is λ :

(1) $\frac{13}{6}$ (2) $\frac{5}{2}$ (3) 2 (4) $\frac{1}{2}$

2. Let the function $f(x)=\left(x^2-1\right)\left|x^2-ax+2\right|+\cos|x|$ be not differentiable at the two points $x=\alpha=2$ and $x=\beta$. Then the distance of the point (α,β) from the line $12x+5y+10=0$ is equal to :

(1) 2 (2) 5 (3) 3 (4) 4

3. Let $f(x)=\int_0^x \mathrm{t}\left(\mathrm{t}^2-9\mathrm{t}+20\right)\mathrm{dt}, 1\le x\le 5$. If the range of f is $[\alpha,\beta]$, then $4(\alpha+\beta)$ equals :

(1) 253 (2) 125 (3) 154 (4) 157

4. Let $\mathrm{A}=\left[\mathrm{a}_{ij}\right]$ be a 2×2 matrix such that $\mathrm{a}_{ij}\in\{0,1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is :

(1) $\frac{1}{4}$ (2) $\frac{5}{8}$ (3) $\frac{3}{8}$ (4) $\frac{3}{4}$

5. Let a straight line L pass through the point $P(2,-1,3)$ and be perpendicular to the lines $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$ and $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is :

(1) $2\sqrt{3}$ (2) 3 (3) 2 (4) $\sqrt{10}$

6. Let P be the foot of the perpendicular from the point $(1,2,2)$ on the line $\mathrm{L}:\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z-2}{2}$. Let the line $\vec{\mathrm{r}}=(-\hat{i}+\hat{j}-2\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k}), \lambda\in\mathbf{R}$, intersect the line L at Q. Then $2(\mathrm{PQ})^2$ is equal to:

(1) 19 (2) 29 (3) 25 (4) 27

7. Let $S = \mathbf{N} \cup \{0\}$. Define a relation R from S to $\mathbf{R}$ by :

$$R = \left\{(x,y) : \log_e y = x \log_e\left(\frac{2}{5}\right), x \in S, y \in \mathbf{R}\right\}.$$

Then, the sum of all the elements in the range of R is equal to :

(1) $\frac{10}{9}$ (2) $\frac{5}{2}$ (3) $\frac{5}{3}$ (4) $\frac{3}{2}$

8. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is $29/45$, then n is equal to :

(1) 6 (2) 4 (3) 5 (4) 3

9. The remainder, when 7^{103} is divided by 23, is equal to :

(1) 9 (2) 14 (3) 17 (4) 6

10. Let $\alpha, \beta (\alpha \neq \beta)$ be the values of m, for which the equations $x+y+z=1; x+2y+4z=m$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^\alpha + n^\beta\right)$ is equal to :

(1) 440 (2) 3080 (3) 3410 (4) 560

11. If $\sin x + \sin^2 x = 1, x \in \left(0, \frac{\pi}{2}\right)$, then

$\left(\cos^{12} x + \tan^{12} x\right) + 3\left(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x\right) + \left(\cos^6 x + \tan^6 x\right)$ is equal to :

(1) 2 (2) 4 (3) 3 (4) 1

12. Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma; \beta, \gamma$ are integers, then the value of $|\beta - \gamma|$ equals.

(1) 18 (2) 15 (3) 33 (4) 27

13. Let a circle C pass through the points $(4,2)$ and $(0,2)$, and its centre lie on $3x+2y+2=0$. Then the length of the chord, of the circle C , whose mid-point is $(1,2)$, is :

(1) $2\sqrt{3}$ (2) $2\sqrt{2}$ (3) $\sqrt{3}$ (4) $4\sqrt{2}$

14. If the domain of the function $\log_5\left(18x - x^2 - 77\right)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 195 (2) 174 (3) 186 (4) 179

15. If for the solution curve $y=f(x)$ of the differential equation $\frac{dy}{dx}+(\tan x)y=\frac{2+\sec x}{(1+2\sec x)^2}$, $x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right), f\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$ (2) $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$ (3) $\frac{5-\sqrt{3}}{2\sqrt{2}}$ (4) $\frac{4-\sqrt{2}}{14}$

16. Let $A=\left[a_{ij}\right]$ be a matrix of order 3×3, with $a_{ij}=(\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha+\beta\sqrt{2},\alpha,\beta\in\mathbf{Z}$, then $\alpha+\beta$ is equal to :

(1) 280 (2) 224 (3) 210 (4) 168

17. Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}-\hat{k}$, and makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i}+\hat{j}+\hat{k}$. If a makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i}+\alpha\hat{j}+\hat{k}$, then the value of α is:

(1) $\sqrt{6}$ (2) $-\sqrt{3}$ (3) $-\sqrt{6}$ (4) $\sqrt{3}$

18. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440^{th} position in this arrangement, is :

(1) PRNAUK (2) PRNAKU (3) PRKAUN (4) PRKANU

19. If the set of all $a\in\mathbf{R}$, for which the equation $2x^2+(a-5)x+15=3$ a has no real root, is the interval (α,β), and $X=\{x\in Z:\alpha<x<\beta\}$, then $\sum_{x\in X}x^2$ is equal to :

(1) 2119 (2) 2139 (3) 2129 (4) 2109

20. If $\alpha x+\beta y=109$ is the equation of the chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, whose mid point is $\left(\frac{5}{2},\frac{1}{2}\right)$, then $\alpha+\beta$ is equal to :

(1) 72 (2) 46 (3) 58 (4) 37

SECTION-B

21. Let $a_1,a_2,\ldots,a_{2024}$ be an Arithmetic Progression such that $a_1+\left(a_5+a_{10}+a_{15}+\ldots+a_{2020}\right)+a_{2024}=2233$. Then $a_1+a_2+a_3+\ldots+a_{2024}$ is equal to___.

22. Let $y^2 = 12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(SP)(SQ) = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to ____ .

23. Let integers $a, b \in [-3, 3]$ be such that $a + b \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-a}{z+b}\right| = 1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1, z \in C$, where ω and ω^2 are the roots of $x^2 + x + 1 = 0$, is equal to ____ .

24. If $\lim\limits_{t\to 0}\left(\int_0^1 (3x+5)^t dx\right)^{\frac{1}{t}} = \frac{\alpha}{5e}\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then α is equal to_____.

25. If $24\int_0^{\frac{\pi}{4}}\left(\sin\left|4x - \frac{\pi}{12}\right| + [2\sin x]\right)dx = 2\pi + \alpha$, where [•] denotes the greatest integer function, then α is equal to _____ .

NTA ANSWERS

1.	**(3)**	**2.**	**(3)**	**3.**	**(4)**	**4.**	**(3)**	**5.**	**(2)**	**6.**	**(4)**	**7.**	**(3)**
8.	**(1)**	**9.**	**(2)**	**10.**	**(1)**	**11.**	**(1)**	**12.**	**(3)**	**13.**	**(1)**	**14.**	**(3)**
15.	**(4)**	**16.**	**(2)**	**17.**	**(3)**	**18.**	**(3)**	**19.**	**(2)**	**20.**	**(3)**	**21.**	**(11132)**
22.	**(1328)**	**23.**	**(10)**	**24.**	**(64)**	**25.**	**(12)**						