

# JEE (ADVANCED) EXAM-2025

## MATHEMATICS (PAPER-1)

### SECTION – 1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:  
 Full Marks : +3 If **ONLY** the correct option is chosen;  
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);  
 Negative Marks : -1 In all other cases.

1. Let  $\mathbb{R}$  denote the set of all real numbers. Let  $a_i, b_i \in \mathbb{R}$  for  $i \in \{1, 2, 3\}$ .

Define the functions  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$ , and  $h: \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$$

$$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4$$

$$h(x) = f(x+1) - g(x+2)$$

If  $f(x) \neq g(x)$  for every  $x \in \mathbb{R}$ , then the coefficient of  $x^3$  in  $h(x)$  is

- (A) 8 (B) 2  
(C) -4 (D) -6

2. Three students  $S_1, S_2$ , and  $S_3$  are given a problem to solve. Consider the following events:

$U$  : At least one of  $S_1, S_2$ , and  $S_3$  can solve the problem,

$V$  :  $S_1$  can solve the problem, given that neither  $S_2$  nor  $S_3$  can solve the problem,

$W$  :  $S_2$  can solve the problem and  $S_3$  cannot solve the problem,

$T$  :  $S_3$  can solve the problem.

For any event  $E$ , let  $P(E)$  denote the probability of  $E$ . If

$$P(U) = \frac{1}{2}, \quad P(V) = \frac{1}{10}, \quad \text{and} \quad P(W) = \frac{1}{12} \text{ then } P(T) \text{ is equal to}$$

- (A)  $\frac{13}{36}$  (B)  $\frac{1}{3}$   
(C)  $\frac{19}{60}$  (D)  $\frac{1}{4}$

3. Let  $\mathbb{R}$  denote the set of all real numbers. Define the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Then which one of the following statements is TRUE?

- (A) The function  $f$  is NOT differentiable at  $x = 0$
- (B) There is a positive real number  $\delta$ , such that  $f$  is a decreasing function on the interval  $(0, \delta)$
- (C) For any positive real number  $\delta$ , the function  $f$  is NOT an increasing function on the interval  $(-\delta, 0)$
- (D)  $x = 0$  is a point of local minima of  $f$
4. Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Let the transpose of a matrix  $X$  be denoted by  $X^T$ . Then the number of  $3 \times 3$  invertible matrices  $Q$  with integer entries, such that  $Q^{-1} = Q^T$  and  $PQ = QP$  is

- (A) 32 (B) 8 (C) 16 (D) 24

### SECTION – 2 : (Maximum Marks : 12)

- This section contains **THREE (3)** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:  
 Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;  
 Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;  
 Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;  
 Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;  
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);  
 Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;

- choosing ONLY (A) and (B) will get +2 marks;  
 choosing ONLY (A) and (D) will get +2 marks;  
 choosing ONLY (B) and (D) will get +2 marks;  
 choosing ONLY (A) will get +1 mark;  
 choosing ONLY (B) will get +1 mark;  
 choosing ONLY (D) will get +1 mark;  
 choosing no option (i.e. the question is unanswered) will get 0 marks;

5. Let  $L_1$  be the line of intersection of the planes given by the equations

$$2x + 3y + z = 4 \text{ and } x + 2y + z = 5$$

Let  $L_2$  be the line passing through the point  $P(2, -1, 3)$  and parallel to  $L_1$ . Let  $M$  denote the plane given by the equation

$$2x + y - 2z = 6$$

Suppose that the line  $L_2$  meets the plane  $M$  at the point  $Q$ . Let  $R$  be the foot of the perpendicular drawn from  $P$  to the plane  $M$ .

Then which of the following statements is (are) TRUE?

- (A) The length of the line segment PQ is  $9\sqrt{3}$   
 (B) The length of the line segment QR is 15  
 (C) The area of  $\Delta PQR$  is  $\frac{3}{2}\sqrt{234}$   
 (D) The acute angle between the line segments PQ and PR is  $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

6. Let  $\mathbb{N}$  denote the set of all natural numbers, and  $\mathbb{Z}$  denote the set of all integers. Consider the functions  $f: \mathbb{N} \rightarrow \mathbb{Z}$  and  $g: \mathbb{Z} \rightarrow \mathbb{N}$  defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd} \\ (4-n)/2 & \text{if } n \text{ is even} \end{cases}$$

$$\text{and } g(n) = \begin{cases} 3+2n & \text{if } n \geq 0 \\ -2n & \text{if } n < 0 \end{cases}$$

Define  $(g \circ f)(n) = g(f(n))$  for all  $n \in \mathbb{N}$ , and  $(f \circ g)(n) = f(g(n))$  for all  $n \in \mathbb{Z}$ .

Then which of the following statements is (are) TRUE?

- (A)  $g \circ f$  is NOT one-one and  $g \circ f$  is NOT onto  
 (B)  $f \circ g$  is NOT one-one but  $f \circ g$  is onto  
 (C)  $g$  is one-one and  $g$  is onto  
 (D)  $f$  is NOT one-one but  $f$  is onto

7. Let  $\mathbb{R}$  denote the set of all real numbers. Let  $z_1 = 1 + 2i$  and  $z_2 = 3i$  be two complex numbers, where  $i = \sqrt{-1}$ . Let  $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}$

Then which of the following statements is (are) TRUE?

- (A) S is a circle with centre  $\left(-\frac{1}{3}, \frac{10}{3}\right)$
- (B) S is a circle with centre  $\left(\frac{1}{3}, \frac{8}{3}\right)$
- (C) S is a circle with radius  $\frac{\sqrt{2}}{3}$
- (D) S is a circle with radius  $\frac{2\sqrt{2}}{3}$

**SECTION – 3 : (Maximum Marks : 30)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**  
Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;  
Zero Marks : 0 In all other cases.

8. Let the set of all relations  $R$  on the set  $\{a, b, c, d, e, f\}$ , such that  $R$  is reflexive and symmetric, and  $R$  contains exactly 10 elements, be denoted by  $S$ .  
Then the number of elements in  $S$  is \_\_\_\_.
9. For any two points  $M$  and  $N$  in the X Y-plane, let  $\overrightarrow{MN}$  denote the vector from  $M$  to  $N$ , and  $\vec{0}$  denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle  $\Delta PQR$  such that  
$$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}$$
  
Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of  
$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$
 is \_\_\_\_\_.

10. Let  $S$  be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in  $S$ , but 0210222 is **NOT** in  $S$ .

Then the number of elements  $x$  in  $S$  such that at least one of the digits 0 and 1 appears exactly twice in  $x$ , is equal to \_\_\_\_\_.

11. Let  $\alpha$  and  $\beta$  be the real numbers such that  $\lim_{x \rightarrow 0} \frac{1}{x^3} \left( \frac{\alpha}{2} \int_0^x \frac{1}{1-t^2} dt + \beta x \cos x \right) = 2$

Then the value of  $\alpha + \beta$  is \_\_\_\_\_.

12. Let  $\mathbb{R}$  denote the set of all real numbers. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) > 0$  for all  $x \in \mathbb{R}$ , and  $f(x+y) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$ .

Let the real numbers  $a_1, a_2, \dots, a_{50}$  be in an arithmetic progression. If  $f(a_{31}) = 64f(a_{25})$ , and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1) \text{ then the value of } \sum_{i=6}^{30} f(a_i) \text{ is } \underline{\hspace{2cm}}.$$

13. For all  $x > 0$ , let  $y_1(x)$ ,  $y_2(x)$ , and  $y_3(x)$  be the functions satisfying

$$\begin{aligned} \frac{dy_1}{dx} - (\sin x)^2 y_1 &= 0, & y_1(1) &= 5 \\ \frac{dy_2}{dx} - (\cos x)^2 y_2 &= 0, & y_2(1) &= \frac{1}{3} \\ \frac{dy_3}{dx} - \left( \frac{2-x^3}{x^3} \right) y_3 &= 0, & y_3(1) &= \frac{3}{5e} \end{aligned}$$

respectively. Then

$$\lim_{x \rightarrow 0^+} \frac{y_1(x)y_2(x)y_3(x) + 2x}{e^{3x} \sin x}$$

is equal to \_\_\_\_\_.

#### SECTION-4 (Maximum Marks: 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- **List-I** has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

**Full Marks** : +4 ONLY if the option corresponding to the correct combination is chosen;

**Zero Marks** : 0 If none of the options is chosen (i.e. the question is unanswered);

**Negative Marks** : -1 In all other cases.

14. Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	$f_1$	$f_2$	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let  $\alpha$  denote the mean deviation about the mean,  $\beta$  denote the mean deviation about the median, and  $\sigma^2$  denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I**

(P)  $7f_1 + 9f_2$  is equal to

(Q)  $19\alpha$  is equal to

(R)  $19\beta$  is equal to

(S)  $19\sigma^2$  is equal to

**List-II**

(1) 146

(2) 47

(3) 48

(4) 145

(5) 55

(A) (P)  $\rightarrow$  (5) (Q)  $\rightarrow$  (3) (R)  $\rightarrow$  (2) (S)  $\rightarrow$  (4)

(B) (P)  $\rightarrow$  (5) (Q)  $\rightarrow$  (2) (R)  $\rightarrow$  (3) (S)  $\rightarrow$  (1)

(C) (P)  $\rightarrow$  (5) (Q)  $\rightarrow$  (3) (R)  $\rightarrow$  (2) (S)  $\rightarrow$  (1)

(D) (P)  $\rightarrow$  (3) (Q)  $\rightarrow$  (2) (R)  $\rightarrow$  (5) (S)  $\rightarrow$  (4)

15. Let  $\mathbb{R}$  denote the set of all real numbers. For a real number  $x$ , let  $[x]$  denote the greatest integer less than or equal to  $x$ . Let  $n$  denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I**

(P) The minimum value of  $n$  for which the

$$\text{function } f(x) = \left[ \frac{10x^3 - 45x^2 + 60x + 35}{n} \right]$$

is continuous on the interval  $[1, 2]$ , is

(Q) The minimum value of  $n$  for which

$$g(x) = (2n^2 - 13n - 15)(x^3 + 3x),$$

$x \in \mathbb{R}$ , is an increasing function on  $\mathbb{R}$ , is

(R) The smallest natural number  $n$  which is

greater than 5, such that  $x=3$  is a point of

local minima of  $h(x) = (x^2 - 9)^n(x^2 + 2x + 3)$ , is

(S) Number of  $x_0 \in \mathbb{R}$  such that

$$I(x) = \sum_{k=0}^4 \left( \sin \left| x - k \right| + \cos \left| x - k + \frac{1}{2} \right| \right),$$

$x \in \mathbb{R}$ , is **NOT** differentiable at  $x_0$ , is

**List-II**

(1) 8

(2) 9

(3) 5

(4) 6

(5) 10

(A) (P)  $\rightarrow$  (1) (Q)  $\rightarrow$  (3) (R)  $\rightarrow$  (2) (S)  $\rightarrow$  (5)

(B) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (1) (R)  $\rightarrow$  (4) (S)  $\rightarrow$  (3)

(C) (P)  $\rightarrow$  (5) (Q)  $\rightarrow$  (1) (R)  $\rightarrow$  (4) (S)  $\rightarrow$  (3)

(D) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (3) (R)  $\rightarrow$  (1) (S)  $\rightarrow$  (5)

16. Let  $\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$ , and  $\vec{u}$  and  $\vec{v}$  be two vectors, such that  $\vec{u} \times \vec{v} = \vec{w}$  and  $\vec{v} \times \vec{w} = \vec{u}$ . Let  $\alpha, \beta, \gamma$ , and  $t$  be real numbers such that

$$\vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \quad -t\alpha + \beta + \gamma = 0, \quad \alpha - t\beta + \gamma = 0, \quad \text{and} \quad \alpha + \beta - t\gamma = 0$$

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I**

**List-II**

- (P)  $|\vec{v}|^2$  is equal to (1) 0  
 (Q) If  $\alpha = \sqrt{3}$ , then  $\gamma^2$  is equal to (2) 1  
 (R) If  $\alpha = \sqrt{3}$ , then  $(\beta + \gamma)^2$  is equal to (3) 2  
 (S) If  $\alpha = \sqrt{2}$ , then  $t + 3$  is equal to (4) 3  
 (5) 5

- (A) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (1) (R)  $\rightarrow$  (4) (S)  $\rightarrow$  (5)  
 (B) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (4) (R)  $\rightarrow$  (3) (S)  $\rightarrow$  (5)  
 (C) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (1) (R)  $\rightarrow$  (4) (S)  $\rightarrow$  (3)  
 (D) (P)  $\rightarrow$  (5) (Q)  $\rightarrow$  (4) (R)  $\rightarrow$  (1) (S)  $\rightarrow$  (3)

## NTA FINAL ANSWERS

- |                  |          |        |                 |          |
|------------------|----------|--------|-----------------|----------|
| 1. (C)           | 2. (A)   | 3. (C) | 4. (C)          | 5. (A,C) |
| 6. (A,D)         | 7. (A,D) | 8. 105 | 9. 1.15 to 1.25 | 10. 762  |
| 11. 2.35 to 2.45 | 12. 96   | 13. 2  | 14. (C)         | 15. (B)  |
| 16. (A)          |          |        |                 |          |