

**JEE (ADVANCED)-2024****PAPER-1****MATHEMATICS****SECTION 1 (Maximum Marks : 12)**

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +3 If ONLY the correct option is chosen;  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);  
Negative Marks : -1 In all other cases.

1. Let  $f(x)$  be a continuously differentiable function on the interval  $(0, \infty)$  such that  $f(1) = 2$  and

$$\lim_{t \rightarrow x} \frac{t^{10} f(x) - x^{10} f(t)}{t^9 - x^9} = 1 \text{ for each } x > 0. \text{ Then, for all } x > 0, f(x) \text{ is equal to}$$

- (A)  $\frac{31}{11x} - \frac{9}{11}x^{10}$  (B)  $\frac{9}{11x} + \frac{13}{11}x^{10}$  (C)  $\frac{-9}{11x} + \frac{31}{11}x^{10}$  (D)  $\frac{13}{11x} + \frac{9}{11}x^{10}$

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is  $\frac{1}{2}$ . Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is  $\frac{1}{6}$ . Then the probability that the student knows the answer of a randomly chosen question is

- (A)  $\frac{1}{12}$  (B)  $\frac{1}{7}$  (C)  $\frac{5}{7}$  (D)  $\frac{5}{12}$

3. Let  $\frac{\pi}{2} < x < \pi$  be such that  $\cot x = \frac{-5}{\sqrt{11}}$ .

$$\text{Then } \left( \sin \frac{11x}{2} \right) (\sin 6x - \cos 6x) + \left( \cos \frac{11x}{2} \right) (\sin 6x + \cos 6x) \text{ is equal to}$$

(A)  $\frac{\sqrt{11}-1}{2\sqrt{3}}$  (B)  $\frac{\sqrt{11}+1}{2\sqrt{3}}$  (C)  $\frac{\sqrt{11}+1}{3\sqrt{2}}$  (D)  $\frac{\sqrt{11}-1}{3\sqrt{2}}$

4. Consider the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ . Let  $S(p, q)$  be a point in the first quadrant such that  $\frac{p^2}{9} + \frac{q^2}{4} > 1$ .

Two tangents are drawn from  $S$  to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point  $T$  in the fourth quadrant. Let  $R$  be the vertex of the ellipse with positive  $x$ -coordinate and  $O$  be the center of the ellipse. If the area of the triangle  $\triangle ORT$  is  $\frac{3}{2}$ , then which of the following options is correct?

- (A)  $q = 2, p = 3\sqrt{3}$  (B)  $q = 2, p = 4\sqrt{3}$   
(C)  $q = 1, p = 5\sqrt{3}$  (D)  $q = 1, p = 6\sqrt{3}$

**SECTION 2 (Maximum Marks : 12)**

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
FULL MARKS : +4 ONLY if (all) the correct option(s) is(are) chosen;  
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;  
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;  
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);  
Negative Marks : -2 In all other cases.

5. Let  $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ ,  $T_1 = \{(-1 + \sqrt{2})^n : n \in \mathbb{N}\}$  and  $T_2 = \{(1 + \sqrt{2})^n : n \in \mathbb{N}\}$ . Then which of the following statements is (are) TRUE?
- (A)  $Z \cup T_1 \cup T_2 \subset S$
- (B)  $T_1 \cap \left(0, \frac{1}{2024}\right) = \emptyset$ , where  $\emptyset$  denotes the empty set
- (C)  $T_2 \cap (2024, \infty) \neq \emptyset$

(D) For any given  $a, b \in \mathbb{Z}$ ,  $\cos(\pi(a+b\sqrt{2})) + i\sin(\pi(a+b\sqrt{2})) \in \mathbb{Z}$  if and only if  $b = 0$ , where  $i = \sqrt{-1}$

6. Let  $R^2$  denote  $R \times R$ . Let

$$S = \{(a, b, c) : a, b, c \in R \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x, y) \in R^2 - \{(0, 0)\}\}.$$

Then which of the following statements is (are) TRUE?

(A)  $\left(2, \frac{7}{2}, 6\right) \in S$

(B) If  $\left(3, b, \frac{1}{12}\right) \in S$ , then  $|2b| < 1$

(C) For any given  $(a, b, c) \in S$ , the system of linear equations  $ax + by = 1$   $bx + cy = -1$  has a unique solution.

(D) For any given  $(a, b, c) \in S$ , the system of linear equations  $(a+1)x + by = 0$   $bx + (c+1)y = 0$  has a unique solution.

7. Let  $R^3$  denote the three-dimensional space. Take two points  $P = (1, 2, 3)$  and  $Q = (4, 2, 7)$ . Let dist

$(X, Y)$  denote the distance between two points  $X$  and  $Y$  in  $R^3$ . Let

$$S = \{X \in R^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\} \text{ and}$$

$$T = \{Y \in R^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\}$$

Then which of the following statements is (are) TRUE?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

### SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +4 If ONLY the correct integer is entered;  
Zero Marks : 0 In all other cases.

8. Let  $a = 3\sqrt{2}$  and  $b = \frac{1}{5^{\frac{1}{6}}\sqrt{6}}$ . If  $x, y \in R$  are such that  $3x + 2y = \log_a(18)^{\frac{5}{4}}$  and  $2x - y = \log_b(\sqrt{1080})$  then  $4x + 5y$  is equal to \_\_\_\_\_.
9. Let  $f(x) = x^4 + ax^3 + bx^2 + c$  be a polynomial with real coefficients such that  $f(1) = -9$ . Suppose that  $i\sqrt{3}$  is a root of the equation  $4x^3 + 3ax^2 + 2bx = 0$ , where  $i = \sqrt{-1}$ . If  $\alpha_1, \alpha_2, \alpha_3$ , and  $\alpha_4$  are all the roots of the equation  $f(x) = 0$ , then  $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$  is equal to \_\_\_\_\_.
10. Let  $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$ , where  $|A|$  denotes the determinant of  $A$ . Then the number of elements in  $S$  is \_\_\_\_\_.
11. A group of 9 students,  $s_1, s_2, \dots, s_9$ , is to be divided to form three teams X, Y and, Z of sizes 2, 3, and 4, respectively. Suppose that  $s_1$  cannot be selected for the team X, and  $s_2$  cannot be selected for the team Y. Then the number of ways to form such teams, is \_\_\_\_\_.
12. Let  $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$ ,  $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$  and  $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$  be three vectors, where  $\alpha, \beta \in R - \{0\}$  and  $O$  denotes the origin. If  $(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$  and the point  $(\alpha, \beta, 2)$  lies on the plane  $3x + 3y - z + l = 0$ , then the value of  $l$  is \_\_\_\_\_.
13. Let  $X$  be a random variable, and let  $P(X = x)$  denote the probability that  $X$  takes the value  $x$ . Suppose that the points  $(x, P(X = x))$ ,  $x = 0, 1, 2, 3, 4$ , lie on a fixed straight line in the  $xy$ -plane, and  $P(X = x) = 0$  for all  $x \in R - \{0, 1, 2, 3, 4\}$ . If the mean of  $X$  is  $\frac{5}{2}$ , and the variance of  $X$  is  $\alpha$ , then the value of  $24\alpha$  is \_\_\_\_\_.

**SECTION 4 (Maximum Marks : 12)**

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

14. Let  $\alpha$  and  $\beta$  be the distinct roots of the equation  $x^2 + x - 1 = 0$ . Consider the set  $T = \{1, \alpha, \beta\}$ . For a  $3 \times 3$  matrix  $M = (a_{ij})_{3 \times 3}$ , define  $R_i = a_{i1} + a_{i2} + a_{i3}$  and  $C_j = a_{1j} + a_{2j} + a_{3j}$  for  $i = 1, 2, 3$  and  $j = 1, 2, 3$

Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	The number of matrices $M = (a_{ij})_{3 \times 3}$ with all entries in $T$ such that $R_i = C_j = 0$ for all $i, j$ is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3 \times 3}$ with all entries in $T$ such that $C_j = 0$ for all $j$ is	(2)	12
(R)	Let $M = (a_{ij})_{3 \times 3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$ . Then the number of elements in the set $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in R, M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix} \right\}$ is	(3)	Infinite
(S)	Let $M = (a_{ij})_{3 \times 3}$ be a matrix with all entries in $T$ such that $R_i = 0$ for all $i$ . Then the absolute value of the determinant of $M$ is	(4)	6
		(5)	0

The correct option is

- (A) (P)  $\rightarrow$  (4) (Q)  $\rightarrow$  (2) (R)  $\rightarrow$  (5) (S)  $\rightarrow$  (1)  
 (B) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (4) (R)  $\rightarrow$  (1) (S)  $\rightarrow$  (5)  
 (C) (P)  $\rightarrow$  (2) (Q)  $\rightarrow$  (4) (R)  $\rightarrow$  (3) (S)  $\rightarrow$  (5)  
 (D) (P)  $\rightarrow$  (1) (Q)  $\rightarrow$  (5) (R)  $\rightarrow$  (3) (S)  $\rightarrow$  (4)
15. Let the straight line  $y = 2x$  touch a circle with center  $(0, \alpha)$ ,  $\alpha > 0$ , and radius  $r$  at a point  $A_1$ . Let  $B_1$  be the point on the circle such that the line segment  $A_1B_1$  is a diameter of the circle. Let  $\alpha + r = 5 + \sqrt{5}$ . Match each entry in List-I to the correct entry in List-II.

	List - I		List - II
(P)	$\alpha$ equals	(1)	$(-2, 4)$
(Q)	$r$ equals	(2)	$\sqrt{5}$
(R)	$A_1$ equals	(3)	$(-2, 6)$
(S)	$B_1$ equals	(4)	5
		(5)	$(5, 4)$

The correct option is

- (A)  $(P) \rightarrow (4) (Q) \rightarrow (2) (R) \rightarrow (1) (S) \rightarrow (3)$   
 (B)  $(P) \rightarrow (2) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (3)$   
 (C)  $(P) \rightarrow (4) (Q) \rightarrow (2) (R) \rightarrow (5) (S) \rightarrow (3)$   
 (D)  $(P) \rightarrow (2) (Q) \rightarrow (4) (R) \rightarrow (3) (S) \rightarrow (5)$

16. Let  $\gamma \in \mathbb{R}$  be such that the lines  $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$  and  $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$

intersect. Let  $R_1$  be the point of intersection of  $L_1$  and  $L_2$ . Let  $O = (0, 0, 0)$ , and  $\hat{n}$  denote a unit normal vector to the plane containing both the lines  $L_1$  and  $L_2$ .

Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	$\gamma$ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\frac{\sqrt{3}}{\sqrt{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\frac{\sqrt{2}}{\sqrt{3}}$

The correct option is

- (A)  $(P) \rightarrow (3) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (2)$   
 (B)  $(P) \rightarrow (5) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (2)$   
 (C)  $(P) \rightarrow (3) (Q) \rightarrow (4) (R) \rightarrow (1) (S) \rightarrow (5)$   
 (D)  $(P) \rightarrow (3) (Q) \rightarrow (1) (R) \rightarrow (4) (S) \rightarrow (5)$

17. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be functions defined by  $f(x) = \begin{cases} x|x| \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases}$  and

$$g(x) = \begin{cases} 1-2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}$$

Let  $a, b, c, d \in \mathbb{R}$ . Define the function  $h : \mathbb{R} \rightarrow \mathbb{R}$  by

$$h(x) = af(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c(x - g(x)) + dg(x), x \in \mathbb{R}$$

Match each entry in List-I to the correct entry in List-II.

List-I		List-II	
(P)	If $a = 0, b = 1, c = 0$ and $d = 0$ , then	(1)	$h$ is one-one
(Q)	If $a = 1, b = 0, c = 0$ and $d = 0$ , then	(2)	$h$ is onto.
(R)	If $a = 0, b = 0, c = 1$ and $d = 0$ , then	(3)	$h$ is differentiable on $\mathbb{R}$ .
(S)	If $a = 0, b = 0, c = 0$ and $d = 1$ , then	(4)	the range of $h$ is $[0, 1]$
		(5)	the range of $h$ is $\{0, 1\}$

The correct option is

- (A)  $(P) \rightarrow (4) (Q) \rightarrow (3) (R) \rightarrow (1) (S) \rightarrow (2)$   
 (B)  $(P) \rightarrow (5) (Q) \rightarrow (2) (R) \rightarrow (4) (S) \rightarrow (3)$   
 (C)  $(P) \rightarrow (5) (Q) \rightarrow (3) (R) \rightarrow (2) (S) \rightarrow (4)$   
 (D)  $(P) \rightarrow (4) (Q) \rightarrow (2) (R) \rightarrow (1) (S) \rightarrow (3)$

### ANSWER KEY

1. (B) 2. (C) 3. (B) 4. (A) 5. (A, C, D) 6. (B,C,D)  
 7. (A,B,C) 8. 8 9. 20 10. 16 11. 665 12. 5  
 13. 42 14. (C) 15. (C) 16. (C) 17. (C)