

## JEE (ADVANCED)-2024

## PAPER-2

## MATHEMATICS

## SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

**Full Marks** : +3 If **ONLY** the correct option is chosen;

**Zero Marks** : 0 If none of the options is chosen (i.e. the question is unanswered);

**Negative Marks** : 1 In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of  $\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - 2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$  is  
 (A)  $\frac{7}{24}$  (B)  $\frac{-7}{24}$  (C)  $\frac{-5}{24}$  (D)  $\frac{5}{24}$
2. Let  $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x \text{ and } 3y + \sqrt{8}x \leq 5\sqrt{8}\}$ . If the area of the region  $S$  is  $\alpha\sqrt{2}$ , then  $\alpha$  is equal to  
 (A)  $\frac{17}{2}$  (B)  $\frac{17}{3}$  (C)  $\frac{17}{4}$  (D)  $\frac{17}{5}$
3. Let  $k \in \mathbb{R}$ . If  $\lim_{x \rightarrow 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$ , then the value of  $k$  is  
 (A) 1 (B) 2 (C) 3 (D) 4
4. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

- (A)  $f(x) = 0$  has infinitely many solutions in the interval  $\left[\frac{1}{10^{10}}, \infty\right)$ .

- (B)  $f(x) = 0$  has no solutions in the interval  $\left[\frac{1}{\pi}, \infty\right)$ .
- (C) The set of solutions of  $f(x) = 0$  in the interval  $\left(0, \frac{1}{10^{10}}\right)$  is finite.
- (D)  $f(x) = 0$  has more than 25 solutions in the interval  $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$ .

**SECTION 2 (Maximum Marks : 12)**

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;  
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;  
Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen, both of which are correct;  
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;  
Zero Marks : 0 If unanswered;  
Negative Marks : -2 In all other cases.

5. Let  $S$  be the set of all  $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$  such that

$$\lim_{x \rightarrow \infty} \frac{\sin(x^2) (\log_e x)^\alpha \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta} (\log_e(1+x))^\beta} = 0$$

Then which of the following is (are) correct?

- (A)  $(-1, 3) \in S$       (B)  $(-1, 1) \in S$       (C)  $(1, -1) \in S$       (D)  $(1, -2) \in S$
6. A straight line drawn from the point  $P(1, 3, 2)$ , parallel to the line  $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$ , intersects the plane  $L_1 : x - y + 3z = 6$  at the point  $Q$ . Another straight line which passes through  $Q$  and is perpendicular to the plane  $L_1$  intersects the plane  $L_2 : 2x - y + z = -4$  at the point  $R$ . Then which of the following statements is(are) TRUE?

- (A) The length of the line segment PQ is  $\sqrt{6}$
- (B) The coordinates of  $R$  are  $(1, 6, 3)$
- (C) The centroid of the triangle PQR is  $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$
- (D) The perimeter of the triangle PQR is  $\sqrt{2} + \sqrt{6} + \sqrt{11}$
7. Let  $A_1, B_1, C_1$  be three points in the  $xy$ -plane. Suppose that the lines  $A_1C_1$  and  $B_1C_1$  are tangents to the curve  $y^2 = 8x$  at  $A_1$  and  $B_1$ , respectively. If  $O = (0, 0)$  and  $C_1 = (-4, 0)$ , then which of the following statements is (are) TRUE?
- (A) The length of the line segment  $OA_1$  is  $4\sqrt{3}$
- (B) The length of the line segment  $A_1B_1$  is 16
- (C) The orthocentre of the triangle  $A_1B_1C_1$  is  $(0, 0)$
- (D) The orthocentre of the triangle  $A_1B_1C_1$  is  $(1, 0)$

**SECTION 3 (Maximum Marks : 24)**

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +4 If ONLY the correct integer is entered;  
Zero Marks : 0 In all other cases.

8. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$ , and  $g : \mathbb{R} \rightarrow (0, \infty)$  be a function such that  $g(x+y) = g(x)g(y)$  for all  $x, y \in \mathbb{R}$ . If  $f\left(\frac{-3}{5}\right) = 12$  and  $g\left(\frac{-1}{3}\right) = 2$ , then the value of  $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$  is .....

9. A bag contains  $N$  balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For  $i=1,2,3$ , let  $W_i$ ,  $G_i$ , and  $B_i$  denote the events that the ball drawn in the  $i^{\text{th}}$  draw is a white ball, green ball, and blue ball, respectively. If the probability  $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$  and the conditional probability  $P(B_3 | W_1 \cap G_2) = \frac{2}{9}$ , then  $N$  equals.....
10. Let the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by
- $$f(x) = \frac{\sin x (x^{2023} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 3)} + \frac{2 (x^{2023} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 3)}$$
- Then the number of solutions of  $f(x) = 0$  in  $\mathbb{R}$  is
11. Let  $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$  and  $\vec{q} = \hat{i} - \hat{j} + \hat{k}$ . If for some real numbers  $\alpha, \beta$  and  $\gamma$ , we have  $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$ , then the value of  $\gamma$  is
12. A normal with slope  $\frac{1}{\sqrt{6}}$  is drawn from the point  $(0, -\alpha)$  to the parabola  $x^2 = -4ay$ , where  $a > 0$ . Let  $L$  be the line passing through  $(0, -\alpha)$  and parallel to the directrix of the parabola. Suppose that  $L$  intersects the parabola at two points  $A$  and  $B$ . Let  $r$  denote the length of the latus rectum and  $s$  denote the square of the length of the line segment  $AB$ . If  $r : s = 1 : 16$ , then the value of  $24a$  is\_\_\_\_\_
13. Let the function  $f : [1, \infty) \rightarrow \mathbb{R}$  be defined by
- $$f(t) = \begin{cases} (-1)^{n+1} 2, & \text{if } t = 2n-1, n \in \mathbb{N}, \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1, n \in \mathbb{N}. \end{cases}$$
- Define  $g(x) = \int_1^x f(t) dt, x \in (1, \infty)$ . Let  $\alpha$  denote the number of solutions of the equation  $g(x) = 0$  in the interval  $(1, 8]$  and  $\beta = \lim_{x \rightarrow 1+} \frac{g(x)}{x-1}$ . Then the value of  $\alpha + \beta$  is equal to

SECTION 4 (Maximum Marks : 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;  
Zero Marks : 0 In all other cases.

PARAGRAPH - I

Let  $S = \{1, 2, 3, 4, 5, 6\}$  and  $X$  be the set of all relations  $R$  from  $S$  to  $S$  that satisfy both the following properties:

- $R$  has exactly 6 elements.
- For each  $(a, b) \in R$ , we have  $|a - b| \geq 2$ .

Let  $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$  and  $Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$ .

Let  $n(A)$  denote the number of elements in a set  $A$ .

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

14. If  $n(X) = {}^m C_6$ , then the value of  $m$  is

PARAGRAPH - I

Let  $S = \{1, 2, 3, 4, 5, 6\}$  and  $X$  be the set of all relations  $R$  from  $S$  to  $S$  that satisfy both the following properties:

- $R$  has exactly 6 elements.
- For each  $(a, b) \in R$ , we have  $|a - b| \geq 2$ .

Let  $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$  and  $Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$ .

Let  $n(A)$  denote the number of elements in a set  $A$ .

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

15. If the value of  $n(Y) + n(Z)$  is  $k^2$ , then  $|k|$  is \_\_\_\_\_

PARAGRAPH II

Let  $f : \left[0, \frac{\pi}{2}\right] \rightarrow [0, 1]$  be the function defined by  $f(x) = \sin^2 x$  and let  $g : \left[0, \frac{\pi}{2}\right] \rightarrow [0, \infty)$  be the

function defined by  $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$ .

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

16. The value of  $2 \int_0^{\frac{\pi}{2}} f(x)g(x)dx - \int_0^{\frac{\pi}{2}} g(x)dx$  is

PARAGRAPH II

Let  $f : \left[0, \frac{\pi}{2}\right] \rightarrow [0, 1]$  be the function defined by  $f(x) = \sin^2 x$  and let  $g : \left[0, \frac{\pi}{2}\right] \rightarrow [0, \infty)$  be the

function defined by  $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$ .

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

17. The value of  $\frac{16}{\pi^3} \int_0^{\frac{\pi}{2}} f(x)g(x)dx$  is

ANSWER KEY

- |        |        |          |        |          |          |           |
|--------|--------|----------|--------|----------|----------|-----------|
| 1. (B) | 2. (B) | 3. (B)   | 4. (D) | 5. (B,C) | 6. (A,C) | 7. (A, C) |
| 8. 51  | 9. 11  | 10. 01   | 11. 2  | 12. 12   | 13. 5    | 14. 20    |
| 15. 36 | 16. 0  | 17. 0.25 |        |          |          |           |