PAPER_2

SECTION 1 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is

[JEE ADVANCED_2024_P2_ITF]

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$ (C) $\frac{-5}{24}$ (D) $\frac{5}{24}$

Ans. **(B)**

Sol. $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$

Let $\sin^{-1}\frac{3}{5}=\alpha,\quad 2\cos^{-1}\frac{2}{\sqrt{5}}=\beta \quad \Rightarrow \cos\frac{\beta}{2}=\frac{2}{\sqrt{5}}$

$$\because \sin\alpha=\frac{3}{5} \Rightarrow \tan\alpha=\frac{3}{4} \quad \tan\beta=\frac{2\tan\frac{\beta}{2}}{1-\tan^2\frac{\beta}{2}}=\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$$

$$\Rightarrow \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{\frac{3}{4}-\frac{4}{3}}{1+1}=-\frac{7}{24}$$

2. Let $S=\left\{(x,y)\in R\times R : x\ge 0, y\ge 0, y^2\le 4x, y^2\le 12-2x \text{ and } 3y+\sqrt{8}x\le 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

[JEE ADVANCED_2024_P2_AR]

(A) $\frac{17}{2}$ (B) $\frac{17}{3}$ (C) $\frac{17}{4}$ (D) $\frac{17}{5}$

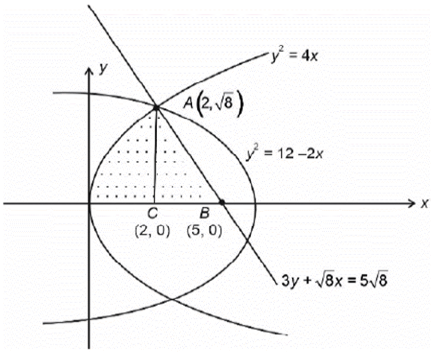
Ans. **(B)**

Sol. $y^2=4x, y^2=12-2x \Rightarrow x=2, y=\sqrt{8}$

A= $\int_0^2 2\sqrt{x}dx + \frac{1}{2}\times 3\times\sqrt{8}$

$$=\left[2\times\frac{2}{3}x^{\frac{3}{2}}\right]_0^2 + 3\sqrt{2} = \frac{4}{3}\times 2\sqrt{2} + 3\sqrt{2} = \frac{17}{3}\sqrt{2}$$

$$\because A = \alpha\sqrt{2} \Rightarrow \alpha = \frac{17}{3}$$



Option (B) is correct.

3. Let $k \in R$. If $\lim_{x\to 0+}(\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

[JEE ADVANCED_2024_P2_LI]

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol. $I = \lim_{x\to 0^+}(\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$

$$\Rightarrow \ln I = \lim_{x\to 0^+}\frac{2}{x}(\sin(\sin kx) + \cos x + x - 1)$$

$$\Rightarrow \ln I = \lim_{x\to 0^+} 2\left(\frac{\sin(\sin kx)}{\sin kx}\cdot\frac{\sin kx}{kx}\cdot\frac{kx}{x} + 1 - \frac{(1-\cos x)}{x^2}\cdot x\right)$$

$$\Rightarrow \ln I = 2(k+1) \Rightarrow I = e^{2(k+1)} = e^6$$

$$k + 1 = 3 \Rightarrow k = 2$$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2\sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

[JEE ADVANCED_2024_P2_FN]

(A) $f(x)=0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}},\infty\right)$.

(B) $f(x)=0$ has no solutions in the interval $\left[\frac{1}{\pi},\infty\right)$.

(C) The set of solutions of $f(x)=0$ in the interval $\left(0,\frac{1}{10^{10}}\right)$ is finite.

(D) $f(x)=0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2},\frac{1}{\pi}\right)$.

Ans. **(D)**

Sol.

$$f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{x^2}\right), & \text{if } x\neq 0,\\ 0, & \text{if } x=0.\end{cases}$$

$$f(x)=0\Rightarrow \sin\left(\frac{\pi}{x^2}\right)=0$$

$$\Rightarrow \frac{\pi}{x^2}=n\pi$$

$$\Rightarrow x^2=\frac{1}{n}$$

$$\Rightarrow x=\frac{1}{\sqrt{n}}$$

If $x\in\left[\frac{1}{10^{10}},\infty\right)$	If $x\in\left[\frac{1}{\pi},\infty\right)$	If $x\in\left(0,\frac{1}{10^{10}}\right)$
$\frac{1}{\sqrt{n}}\in\left[\frac{1}{10^{10}},\infty\right)$	$\frac{1}{\sqrt{n}}\in\left[\frac{1}{\pi},\infty\right)$	$\sqrt{n}\in\left(10^{10},\infty\right)$
$\sqrt{n}\in\left(0,10^{10}\right]$	$\sqrt{n}\in(0,\pi]$	n infinite
$n\in\left(0,\left(10^{10}\right)^2\right]$	$n\in\left(0,\pi^2\right]$	If $x\in\left(\frac{1}{\pi^2},\frac{1}{\pi}\right)$
Finite values of n	$n=1,2,3\ldots 9$	$\sqrt{n}\in\left(\pi,\pi^2\right)$

$$n\in\left(\pi^2,\pi^4\right)$$

$$n\in(9.8,97.2\ldots)$$

More than 25 solutions

SECTION 2 (Maximum Marks : 12)

• This section contains THREE (03) questions.

• Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are)

correct answer(s).

• For each question, choose the option(s) corresponding to (all) the correct answer(s).

• Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks : –2 In all other cases.

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5. Let S be the set of all $(\alpha,\beta)\in R\times R$ such that

$$\lim_{x\to\infty}\frac{\sin\left(x^2\right)\left(\log_e x\right)^{\alpha}\sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}\left(\log_e(1+x)\right)^{\beta}}=0$$

Then which of the following is (are) correct?

[JEE ADVANCED_2024_P2_LI]

(A) $(-1,3)\in S$

(B) $(-1,1)\in S$

(C) $(1,-1)\in S$

(D) $(1,-2)\in S$

Ans. **(B,C)**

Sol.

$$\lim_{x\to\infty}\frac{\sin\left(x^2\right)\sin\left(\frac{1}{x^2}\right)(\ln x)^{\alpha}}{x^{\alpha\beta}(\ln(1+x))^{\beta}}=0$$

$$=\lim_{x\to\infty}\frac{\left(\sin x^2\right)\sin\left(\frac{1}{x^2}\right)\frac{1}{x^2}}{\left(\frac{1}{x^2}\right)x^{\alpha\beta}(\ln(1+x))^{\beta}}=0$$

It is possible if $\alpha\beta + 2 > 0$ $\alpha\beta > -2$

(A) $\alpha\beta = -3$ (B) $\alpha\beta = -1$ (C) $\alpha\beta = -1$ (D) $\alpha\beta = -2$

6. A straight line drawn from the point $P(1,3,2)$, parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R. Then which of the following statements is(are) TRUE?

[JEE ADVANCED_2024_P2_3D]

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are $(1,6,3)$

(C) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$

Ans. **(A,C)**

Sol. Equation of line parallel to $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$ through $P(1,3,2)$ is

$\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-2}{1} = \lambda$ (let)

Now, putting any point $(\lambda+1, 2\lambda+3, \lambda+2)$ in L_1

$\boxed{\lambda = 1}$

$\Rightarrow$ Point $Q(2,5,3)$

Equation of line through $Q(2,5,3)$ perpendicular to L_1 is

$\frac{x-2}{1} = \frac{y-5}{-1} = \frac{z-3}{3} = \mu$ (Let)

Putting any point $(\mu+2, -\mu+5, 3\mu+3)$ in L_2

$\mu = -1$

$\Rightarrow$ Point $R(1,6,0)$

(A) $PQ = \sqrt{1+4+1} = \sqrt{6}$

(B) $R(1,6,0)$

(C) Centroid $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(D) $PQ+QR+PR=\sqrt{6}+\sqrt{11}+\sqrt{13}$

7. Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2=8x$ at A_1 and B_1, respectively. If $O=(0,0)$ and $C_1=(-4,0)$, then which of the following statements is (are) TRUE?

[JEE ADVANCED_2024_P2_PB]

(A) The length of the line segment OA_1 is $4\sqrt{3}$

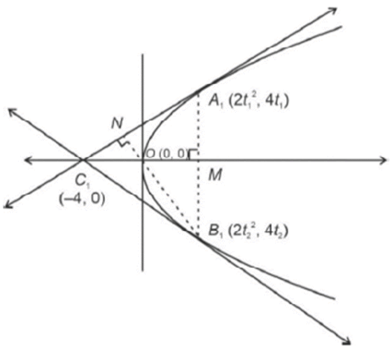
(B) The length of the line segment A_1B_1 is 16

(C) The orthocentre of the triangle $A_1B_1C_1$ is $(0,0)$

(D) The orthocentre of the triangle $A_1B_1C_1$ is $(1,0)$

Ans. **(A, C)**

Sol.



Let $A_1=\left(2t_1^2, 4t_1\right)$ and $B_1=\left(2t_2^2, 4t_2\right)$

$C \equiv (-4,0) \equiv \left(2t_1t_2, 2\left(t_1+t_2\right)\right)$

$\Rightarrow t_2=-t_1 \text{ and } t_1\left(-t_1\right)=-2$

$t_1=\sqrt{2}, t_2=-\sqrt{2}$

$A_1 \equiv (4, 4\sqrt{2}), B_1 \equiv (4, -4\sqrt{2})$

$\therefore \quad OA_1=\sqrt{4^2+(4\sqrt{2})^2}=4\sqrt{3}$

$A_1B_1=8\sqrt{2}$

Altitude $C_1M: y=0$

Altitude $B_1N: \sqrt{2}x+y=0$

$\therefore$ Orthocentre $\equiv (0,0)$

SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

8. Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in R$, and $g : R \to (0, \infty)$ be a function such that $g(x+y) = g(x)g(y)$ for all $x, y \in R$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is ....

[JEE ADVANCED_2024_P2_C&D]

Ans. **(51)**

Sol. $f(x+y) = f(x) + f(y)$

$\Rightarrow f(x) = kx$

$$f\left(\frac{-3}{5}\right) = 12 \Rightarrow k = -20$$

$\therefore \quad f(x) = -20x$

$$g(x+y) = g(x)g(y) \Rightarrow g(x) = a^x$$

$$g\left(\frac{-1}{3}\right) = 2 \Rightarrow a = \frac{1}{8}$$

$$\therefore \quad g(x) = \left(\frac{1}{8}\right)^x$$

$$\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0) = (-5 + 64 - 8) \times 1 = 51$$

9. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For i=1,2,3, let W_i, G_i, and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively, If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 | W_1 \cap G_2) = \frac{2}{9}$, then N equals..........

[JEE ADVANCED_2024_P2_PB]

Ans. (11)

Sol. N Balls $= 3W + 6G + (N-9)B$

$$P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$$

$$\Rightarrow \frac{3}{N} \times \frac{6}{N-1} \times \frac{N-9}{N-2} = \frac{2}{5N}$$

$$\Rightarrow N^2 - 48N + 407 = 0$$

N = 11 or 37

$$P(B_3 | W_1 \cap G_2) = \frac{2}{9}$$

$$\Rightarrow \frac{P(W_1 \cap G_2 \cap B_3)}{P(W_1 \cap G_2)} = \frac{2}{9}$$

$$\Rightarrow \frac{\frac{2}{5N}}{\frac{3}{N} \times \frac{6}{N-1}} = \frac{2}{9}$$

$$\Rightarrow \frac{N-1}{45} = \frac{2}{9}$$

$$\Rightarrow N = 11$$

10. Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}$$

Then the number of solutions of $f(x) = 0$ in R is

[JEE ADVANCED_2024_P2_MO](AOD me dale hai ex. 3 me)

Ans. (01)

Sol. $f(x) = 0$

$$\Rightarrow \frac{x^{2023} + 2024x + 2025}{(x^2 - x + 3)} \left[\frac{\sin x + 2}{e^{\pi x}} \right] = 0$$

$$\Rightarrow x^{2023} + 2024x + 2025 = 0$$

Let $g(x)=x^{2023}+2024x+2025$

$g'(x)=2023x^{2022}+2024>0\forall x\in R$

$\therefore$ f(x)=0 has only one solution

11. Let $\vec{p}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{q}=\hat{i}-\hat{j}+\hat{k}$. If for some real numbers α,β and γ, we have $15\hat{i}+10\hat{j}+6\hat{k}=\alpha(2\vec{p}+\vec{q})+\beta(\vec{p}-2\vec{q})+\gamma(\vec{p}\times\vec{q})$, then the value of γ is

[JEE ADVANCED_2024_P2_VE]

Ans. **(2)**

Sol. $2\vec{p}+\vec{q}=5\hat{i}+\hat{j}+7\hat{k}$

$\vec{p}-2\vec{q}=0\hat{i}+3\hat{j}+\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 3\\ 1 & -1 & 1\end{vmatrix}$$

$=\hat{i}(4)-\hat{j}(-1)+\hat{k}(-3)$

$=4\hat{i}+\hat{j}-3\hat{k}$

$15\hat{i}+10\hat{j}+6\hat{k}=\alpha(5\hat{i}+\hat{j}+7\hat{k})+\beta(3\hat{j}+\hat{k})+\gamma(4\hat{i}+\hat{j}-3\hat{k})$

$\therefore\ 15=5\alpha+4\gamma$

$10=\alpha+3\beta+\gamma$

$6=7\alpha+\beta-3\gamma$

$\therefore\ \alpha=\frac{7}{5},\beta=\frac{11}{5},\gamma=2$

$\therefore\ \gamma=2$

12. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0,-\alpha)$ to the parabola $x^2=-4ay$, where $a>0$. Let L be the line passing through $(0,-\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r:s=1:16$, then the value of 24a is____

[JEE ADVANCED_2024_P2_PB]

Ans. **(12)**

Sol. $x^2 = -4ay$

Equation of normal

$$y = mx - 2a - \frac{a}{m^2}$$

$$-\alpha = -2a - \frac{a}{\frac{1}{6}} = -8a$$

$$\Rightarrow \alpha = 8a$$

Equation of required line

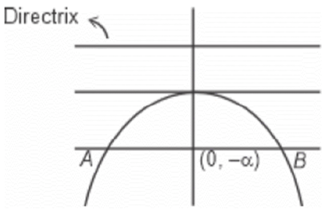
$$y = -\alpha$$

$$\Rightarrow y = -8a, \text{ solving with } x^2 = -4ay$$

$$\Rightarrow x^2 = 32a^2$$

$$\Rightarrow x = \pm 4\sqrt{2}a$$

$$= \pm\frac{\alpha}{\sqrt{2}}$$



$$A\left(\frac{\alpha}{\sqrt{2}}, -\alpha\right), B\left(\frac{-\alpha}{\sqrt{2}}, -\alpha\right) \Rightarrow AB = \sqrt{2}\alpha$$

$$\Rightarrow \frac{r}{s} = \frac{4a}{2\alpha^2} = \frac{1}{16} \Rightarrow \frac{4a}{2 \times 64a^2} = \frac{1}{16}$$

$$\Rightarrow a = \frac{1}{2}$$

$$\Rightarrow \boxed{24a = 12}$$

13. Let the function $f : [1, \infty) \to R$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1}2, & \text{if } t = 2n-1, n \in N, \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1, n \in N. \end{cases}$$

Define $g(x)=\int_1^x f(t)dt, x\in(1,\infty)$. Let α denote the number of solutions of the equation $g(x)=0$ in the interval

$(1,8]$ and $\beta=\lim_{x\to 1+}\frac{g(x)}{x-1}$. Then the value of $\alpha+\beta$ is equal to

[JEE ADVANCED_2024_P2_DI]

Ans. (5)

Sol. $f(t)=\left(\frac{(2n+1)-t}{2}\right)(-1)^{n+1}2+\left(\frac{t-(2n-1)}{2}\right)(-1)^{n+2}2, t\in(2n-1,2n+1)$

$$\Rightarrow f(t)=2(-1)^{n+1}(2n-t), t\in(2n-1,2n+1)$$

$$\Rightarrow g(x)=\int_1^x f(t)dt, x\in(1,8]$$

$$=\begin{cases}\int_1^x 2(2-t)dt, 1<x\le 3, n=1 \\ 3 \\ \int_1^3 2(2-t)dt+\int_3^x(2t-8)dt, 3<x\le 5, n=2 \\ 3 \\ \int_1^3 2(2-t)dt+\int_3^5(2t-8)dt+\int_5^x 2(6-t)dt, 5<x\le 7, n=3 \\ 3 \\ \int_1^3 2(2-t)dt+\int_3^5(2t-8)dt+\int_5^7 2(6-t)dt+\int_7^x(2t-16)dt, x\in(7,8], n=4\end{cases}$$

$$=\begin{cases}-x^2+4x-3, 1<x\le 3, \\ x^2-8x+15, 3<x\le 5 \\ -x^2+12x-35, 5<x\le 7 \\ x^2-16x+63, 7<x\le 8\end{cases}=\begin{cases}-(x-1)(x-3), 1<x\le 3 \\ (x-3)(x-5), 3<x\le 5 \\ -(x-5)(x-7), 5<x\le 7 \\ (x-7)(x-9), 7<x\le 8\end{cases}$$

$$\Rightarrow g(x)=0\Rightarrow x=3,5,7\Rightarrow \alpha=3$$

$$\beta=\lim_{x\to 1^+}\left(\frac{g(x)}{x-1}\right)=\lim_{x\to 1^+}-\frac{(x-1)(x-3)}{x-1}=2$$

$$\Rightarrow \alpha+\beta=5$$

SECTION 4 (Maximum Marks : 12)

- This section contains TWO (02) paragraphs.
- Based on each paragraph, there are TWO (02) questions.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual

numeric keypad in the place designated to enter the answer.

• If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

• Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

PARAGRAPH - I

Let $S=\{1,2,3,4,5,6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a,b)\in R$, we have $|a-b|\geq 2$.

Let $Y=\{R\in X$: The range of R has exactly one element $\}$ and $Z=\{R\in X: R$ is a function from S to $S\}$.

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

[JEE ADVANCED_2024_P2_FN]

14. If $n(X)={}^{m}C_6$, then the value of m is

Ans. **(20)**

Sol. $S=\{1,2,3,4,5,6\}$ $\quad R:S\to S$

Number of elements in $R=6$ and for each $(a,b)\in R; |a-b|\geq 2$

$X\to$ set of all relation $R:S\to S$

If

$a=1, b=3,4,5,6 \to ④$
$a=2, b=4,5,6 \to ③$
$a=3, b=1,5,6 \to ③$
$a=4, b=1,2,6 \to ③$
$a=5, b=1,2,3 \to ③$
$a=6, b=1,2,3,4 \to ④$

Total number of ordered pairs (a, b) s.t. $|a-b|\geq 2$ $= 20$

$\therefore n(X)=$ number of elements in X

$={}^{20}C_6 \quad \therefore m=20$

PARAGRAPH - I

Let $S=\{1,2,3,4,5,6\}$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in R$, we have $|a-b| \geq 2$.

Let $Y=\{R \in X$: The range of R has exactly one element $\}$ and $Z=\{R \in X: R$ is a function from S to $S\}$.

Let $n(A)$ denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

[JEE ADVANCED_2024_P2_FN]

15. If the value of $n(Y)+n(Z)$ is k^2, then |k| is _____

Ans **(36)**

Sol. $S=\{1,2,3,4,5,6\}$ $\quad\quad R: S \rightarrow S$

Number of elements in R = 6 and for each $(a, b) \in R; |a-b| \geq 2$

$X \rightarrow$ set of all relation $R: S \rightarrow S$

If

$a=1$	$b=3,4,5,6$	$\rightarrow$	4
$a=2$	$b=4,5,6$	$\rightarrow$	3
$a=3$	$b=1,5,6$	$\rightarrow$	3
$a=4$	$b=1,2,6$	$\rightarrow$	3
$a=5$	$b=1,2,3$	$\rightarrow$	3
$a=6$	$b=1,2,3,4$	$\rightarrow$	4

} Total number of ordered pairs (a, b) s. t. $|a-b| \geq 2 = 20$

$\therefore \quad n(X)=$ number of elements in X

$={}^{20}C_6 \quad \therefore m=20$

$Y=\{R \in X$: The range of R has exactly one element$\}$

From above, if range of R has exactly one element, then maximum number of elements in R will be 4 .

$\therefore \quad n(Y)=0$

$Z=\{R \in X: R$ is a function from S to $S\}$

$$n(Z)={}^4C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^4C_1$$

$$=(36)^2$$

$$n(y)+n(z)=0+(36)^2=k^2$$

$\Rightarrow \quad |k|=36$

PARAGRAPH II

Let $f:\left[0,\frac{\pi}{2}\right]\rightarrow[0,1]$ be the function defined by $f(x)=\sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\rightarrow[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

[JEE ADVANCED_2024_P2_DI]

16. The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx-\int_0^{\frac{\pi}{2}} g(x)dx$ is

Ans. **(0)**

Sol. $f(x)=\sin^2 x, g(x)=\sqrt{\frac{\pi}{2}x-x^2}$

Here $f\left(\frac{\pi}{2}-x\right)=\cos^2 x, g\left(\frac{\pi}{2}-x\right)=g(x)$

Let $I_1=2\int_0^{\frac{\pi}{2}} f(x)g(x)=2\int_0^{\frac{\pi}{2}}\sin^2 x\cdot g(x)dx$ ...(1)

as $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$\Rightarrow I_1=2\int_0^{\frac{\pi}{2}}\cos^2 x g(x)dx$ ...(2)

$(1)+(2)$

$\Rightarrow 2I_1=2\int_0^{\frac{\pi}{2}} g(x)dx$

$\Rightarrow I_1=\int_0^{\frac{\pi}{2}} g(x)dx$

PARAGRAPH II

Let $f:\left[0,\frac{\pi}{2}\right]\rightarrow[0,1]$ be the function defined by $f(x)=\sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\rightarrow[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

[JEE ADVANCED_2024_P2_DI]

17. The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is

Ans. **(0.25)**

Sol. According to Q .16

$$2\int_0^{\frac{\pi}{2}} f(x)g(x)dx = \int_0^{\frac{\pi}{2}} g(x)dx = I_1 \text{ (let)}$$

Now, $I_1 = \int_0^{\frac{\pi}{2}} g(x)dx = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\pi}{2}x - x^2}\,dx$

$$I_1 = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{\pi}{4}\right)^2 - \left(\frac{\pi}{4} - x\right)^2}$$

Put $\frac{\pi}{4} - x = t$

$\Rightarrow dx = -dt$

$$I_1 = -\int_{\frac{\pi}{4}}^{\frac{-\pi}{4}} \sqrt{\left(\frac{\pi}{4}\right)^2 - t^2}\,dt$$

$$I_1 = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sqrt{\left(\frac{\pi}{4}\right)^2 - t^2}\,dt$$

$$I_1 = 2\int_0^{\frac{\pi}{4}} \sqrt{\left(\frac{\pi}{4}\right)^2 - t^2}\,dt = 2\left[\frac{t}{2}\sqrt{\left(\frac{\pi}{4}\right)^2 - t^2} + \frac{\pi^2}{32}\sin^{-1}\left(\frac{4t}{\pi}\right)\right]_0^{\frac{\pi}{4}}$$

$$I_1 = \frac{\pi^3}{32}$$

Now, $I = \frac{8}{\pi^3} I_1$

$$I = \frac{1}{4} = 0.25$$