

**JEE-MAIN EXAM APRIL, 2025**

Date: - 03-04-2025 (SHIFT-1)

**MATHEMATICS**

**SECTION-A**

1. Let  $\alpha$  and  $\beta$  be the roots of  $x^2 + \sqrt{3}x - 16 = 0$ , and  $\gamma$  and  $\delta$  be the roots of  $x^2 + 3x - 1 = 0$ . If  $P_n = \alpha^n + \beta^n$  and  $Q_n = \gamma^n + \delta^n$ , then  $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$  is equal to  
 (1) 4                                      (2) 7                                      (3) 3                                      (4) 5
  
2. The radius of the smallest circle which touches the parabolas  $y = x^2 + 2$  and  $x = y^2 + 2$  is  
 (1)  $\frac{7\sqrt{2}}{2}$                                       (2)  $\frac{7\sqrt{2}}{8}$   
 (3)  $\frac{7\sqrt{2}}{16}$                                       (4)  $\frac{7\sqrt{2}}{4}$
  
3. Let  $a_1, a_2, a_3, \dots$  be a G.P. of increasing positive numbers. If  $a_3 a_5 = 729$  and  $a_2 + a_4 = \frac{111}{4}$ , then  $24(a_1 + a_2 + a_3)$  is equal to  
 (1) 131                                      (2) 129                                      (3) 128                                      (4) 130
  
4. If  $\sum_{r=1}^9 \left(\frac{r+3}{2^r}\right) \cdot C_r = \alpha \left(\frac{3}{2}\right)^9 - \beta, \alpha, \beta \in \mathbb{Q}$ , then  $(\alpha + \beta)^2$  is equal to  
 (1) 9                                      (2) 81                                      (3) 18                                      (4) 27
  
5. Let a line passing through the point  $(4, 1, 0)$  intersect the line  $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  at the point  $A(\alpha, \beta, \gamma)$  and the line  $L_2 : x - 6 = y = -z + 4$  at the point  $B(a, b, c)$ . Then  $\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$  is equal to  
 (1) 6                                      (2) 8                                      (3) 16                                      (4) 12
  
6. Let  $A = \{-3, -2, -1, 0, 1, 2, 3\}$ . Let  $R$  be a relation on  $A$  defined by  $xRy$  if and only if  $0 \leq x^2 + 2y \leq 4$ . Let  $l$  be the number of elements in  $R$  and  $m$  be the minimum number of elements required to be added in  $R$  to make it a reflexive relation. Then  $l + m$  is equal to  
 (1) 17                                      (2) 18                                      (3) 20                                      (4) 19

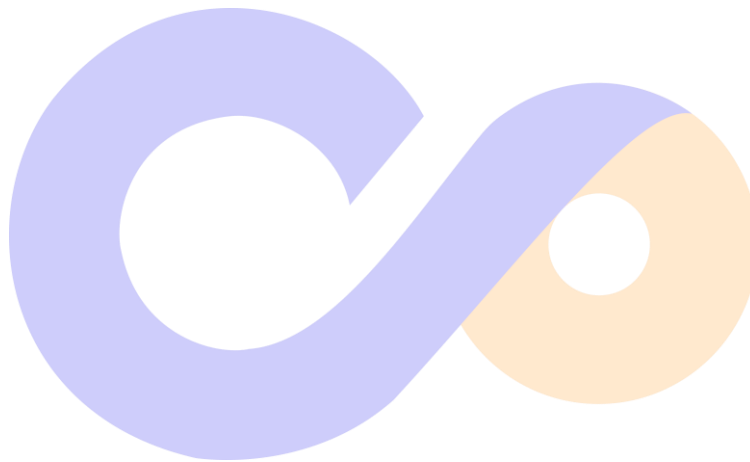
7. A line passing through the point  $P(\sqrt{5}, \sqrt{5})$  intersects the ellipse  $\frac{x^2}{36} + \frac{y^2}{25} = 1$  at  $A$  and  $B$  such that  $(PA) \cdot (PB)$  is maximum. Then  $5(PA^2 + PB^2)$  is equal to :
- (1) 338                      (2) 218                      (3) 377                      (4) 290
8. Line  $L_1$  passes through the point  $(1, 2, 3)$  and is parallel to  $z$ -axis. Line  $L_2$  passes through the point  $(\lambda, 5, 6)$  and is parallel to  $y$ -axis. Let for  $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$ , the shortest distance between the two lines be 3. Then the square of the distance of the point  $(\lambda_1, \lambda_2, 7)$  from the line  $L_1$  is
- (1) 37                      (2) 32                      (3) 40                      (4) 25
9. Let  $g$  be a differentiable function such that  $\int_0^x g(t)dt = x - \int_0^x tg(t)dt, x \geq 0$  and let  $y = y(x)$  satisfy the differential equation  $\frac{dy}{dx} - y \tan x = 2(x+1) \sec x g(x), x \in \left[0, \frac{\pi}{2}\right)$ . If  $y(0) = 0$ , then  $y\left(\frac{\pi}{3}\right)$  is equal to
- (1)  $\frac{2\pi}{3}$                       (2)  $\frac{4\pi}{3}$   
 (3)  $\frac{2\pi}{3\sqrt{3}}$                       (4)  $\frac{4\pi}{3\sqrt{3}}$
10. If the domain of the function  $f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$  is  $[\alpha, \beta]$ , then  $\alpha^2 + 4\beta$  is equal to
- (1) 3                      (2) 7                      (3) 5                      (4) 4
11. Let  $A$  be a matrix of order  $3 \times 3$  and  $|A| = 5$ . If  $|2 \text{adj}(3A \text{adj}(2A))| = 2^\alpha \cdot 3^\beta \cdot 5^\gamma, \alpha, \beta, \gamma \in \mathbb{N}$ , then  $\alpha + \beta + \gamma$  is equal to
- (1) 28                      (2) 27                      (3) 26                      (4) 25
12. If  $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}, x \in \mathbb{R}$ , then  $\frac{d^2y}{dx^2} + y$  is equal to
- (1) 27                      (2) 28                      (3) 1                      (4) -1
13. Let  $z \in \mathbb{C}$  be such that  $\frac{z^2 + 3i}{z - 2 + i} = 2 + 3i$ . Then the sum of all possible values of  $z^2$  is
- (1)  $-19 + 2i$                       (2)  $19 - 2i$                       (3)  $-19 - 2i$                       (4)  $19 + 2i$
14. The number of solutions of the equation  $2x + 3 \tan x = \pi, x \in [-2\pi, 2\pi] - \left\{\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}\right\}$  is:
- (1) 4                      (2) 3                      (3) 5                      (4) 6

15. Let  $f(x) = \int x^3 \sqrt{3-x^2} dx$ . If  $5f(\sqrt{2}) = -4$ , then  $f(1)$  is equal to
- (1)  $-\frac{2\sqrt{2}}{5}$  (2)  $-\frac{8\sqrt{2}}{5}$   
 (3)  $-\frac{6\sqrt{2}}{5}$  (4)  $-\frac{4\sqrt{2}}{5}$
16. The sum of all rational terms in the expansion of  $(2 + \sqrt{3})^8$  is
- (1) 16923 (2) 33845 (3) 3763 (4) 18817
17. Let the domain of the function  $f(x) = \log_2 \log_4 \log_6 (3 + 4x - x^2)$  be  $(a, b)$ . If  $\int_0^{b-a} [x^2] dx = p - \sqrt{q} - \sqrt{r}$ ,  $p, q, r \in \mathbb{Q}$ ,  $\gcd(p, q, r) = 1$ , where  $[\cdot]$  is the greatest integer function, then  $p + q + r$  is equal to
- (1) 11 (2) 9 (3) 8 (4) 10
18. The sum  $1 + 3 + 11 + 25 + 45 + 71 + \dots$  upto 20 terms, is equal to
- (1) 6982 (2) 8124 (3) 7130 (4) 7240
19. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines  $L_1 : 2x + y + 6 = 0$  and  $L_2 : 4x + 2y - p = 0, p > 0$ , at the points A and B, respectively. If  $AB = \frac{9}{\sqrt{2}}$  and the foot of the perpendicular from the point A on the line  $L_2$  is M, then  $\frac{AM}{BM}$  is equal to
- (1) 5 (2) 4 (3) 2 (4) 3
20. Let  $f(x) = \begin{cases} (1+ax)^{1/x}, & x < 0 \\ 1+b, & x = 0 \\ \frac{(x+4)^{1/2} - 2}{(x+c)^{1/3} - 2}, & x > 0 \end{cases}$  be continuous at  $x = 0$ . Then  $e^a bc$  is equal to:
- (1) 72 (2) 48 (3) 36 (4) 64

**SECTION-B**

21. All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number  $n$  be denoted by  $W_n$ . Let the probability  $P(W_n)$  of choosing the word  $W_n$  satisfy  $P(W_n) = 2P(W_{n-1}), n > 1$ .
- If  $P(\text{CDBEA}) = \frac{2^\alpha}{2^\beta - 1}, \alpha, \beta \in \mathbb{Q}$ , then  $\alpha + \beta$  is equal to : \_\_\_\_\_

22. Let the product of the focal distances of the point  $P(4, 2\sqrt{3})$  on the hyperbola  $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be 32 .  
 Let the length of the conjugate axis of H be  $p$  and the length of its latus rectum be  $q$  . Then  $p^2 + q^2$  is equal to \_\_\_\_\_
23. If the number of seven-digit numbers, such that the sum of their digits is even, is  $m \cdot n \cdot 10^n; m, n \in \{1, 2, 3, \dots, 9\}$ , then  $m + n$  is equal to \_\_\_\_\_
24. Let  $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \lambda\hat{j} + \mu\hat{k}$  and  $\hat{d}$  be a unit vector such that  $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$  and  $\vec{c} \cdot \hat{d} = 1$  . If  $\vec{c}$  is perpendicular to  $\vec{a}$  , then  $|3\lambda\hat{d} + \mu\vec{c}|^2$  is equal to \_\_\_\_\_.
25. The area of the region bounded by the curve  $y = \max\{|x|, |x - 2|\}$ , the  $x$ -axis and the lines  $x = -2$  and  $x = 4$  is equal to \_\_\_\_\_.



**NTA ANSWERS**

- |     |       |     |      |     |     |     |      |     |     |     |     |     |       |
|-----|-------|-----|------|-----|-----|-----|------|-----|-----|-----|-----|-----|-------|
| 1.  | (4)   | 2.  | (2)  | 3.  | (2) | 4.  | (2)  | 5.  | (2) | 6.  | (2) | 7.  | (1)   |
| 8.  | (4)   | 9.  | (2)  | 10. | (4) | 11. | (2)  | 12. | (4) | 13. | (3) | 14. | (3)   |
| 15. | (3)   | 16. | (4)  | 17. | (4) | 18. | (4)  | 19. | (4) | 20. | (2) | 21. | (183) |
| 22. | (120) | 23. | (14) | 24. | (5) | 25. | (12) |     |     |     |     |     |       |