NSEP 2026

Held On 22 November, Sunday 2026	TIME : 8 : 30 AM to 10 : 30 AM

PHYSICS

PART A-1

Question 1 (Part A-1, single correct, +3 / -1)

A spherical bob of mass 0.20 kg and diameter 2.0 cm, having uniform density, is attached to the end of a string of length 100 cm. This pendulum so formed is suspended from the ceiling of an elevator. The elevator is moving upward with an acceleration of 5.4 ms^{-2} along an incline which makes an angle of 60 degree with vertical. The pendulum is made to oscillate about its mean position. Assuming that there is no aerial friction, the time period of oscillation of the pendulum is (given that acceleration due to gravity is 9.8 ms^{-2})

(a) 1.28 s (b) 1.26 s (c) 1.59 s (d) 1.79 s

Solution

Correct option: D. **Method:** The effective gravity is $g_{eff} = \sqrt{(g^2 + a^2 + 2ga\cos 60°)} = \sqrt{(9.8^2 + 5.4^2 + 9.8 \times 5.4)} = 13.34\ ms^{-2}$. The effective length is L = 100 + 1 = 101 cm = 1.01 m (string plus bob radius). $T = 2\pi\sqrt{(L/g_{eff})} = 2 \times 3.14 \times \sqrt{(1.01/13.34)} = 1.72$ s. The closest option is (d) 1.79 s.

Answer: (D).

Question 2 (Part A-1, single correct, +3 / -1)

When light is incident on an interface between two media at an angle greater than $\theta_c = \sin^{-1}(1/\mu)$ (where μ is the relative refractive index of the two media), it is totally reflected, a phenomenon known as total internal reflection. If the angle of incidence is smaller than θ_c light is partly reflected and partly refracted. A ray of light incident, parallel to the base, on a 30° - 90° - 60° prism, as shown, suffers two total internal reflections before emerging parallel but reversed in direction. The condition on the critical angle for this to be possible is: Figure: a right-angled prism ABC with the right angle at B, 60° at A and 30° at C. A ray of light comes from the right, parallel to the base BC, and strikes the hypotenuse AC.

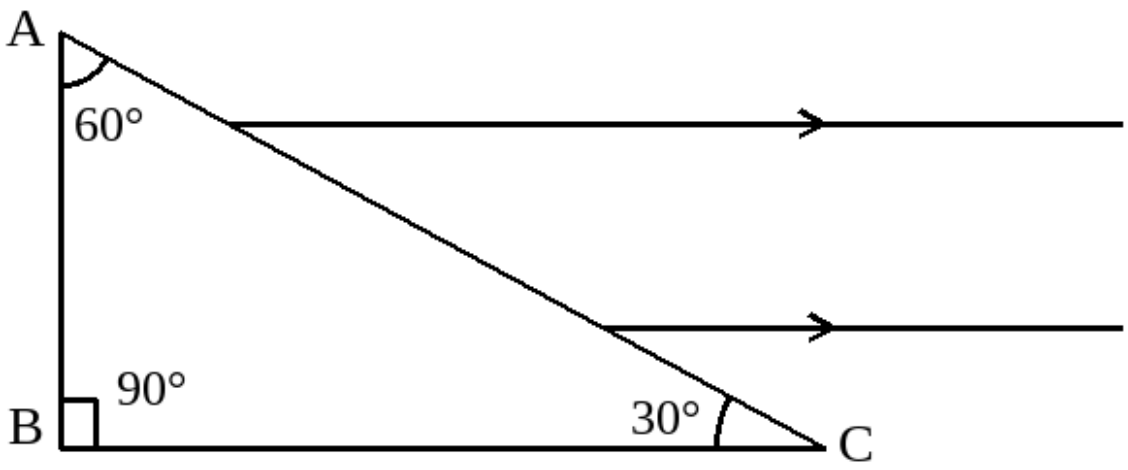
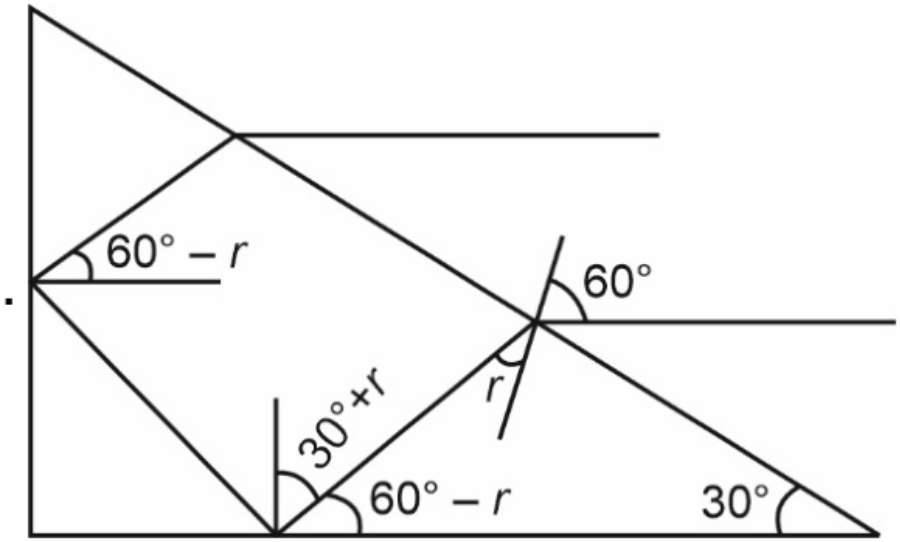


Figure for Question 2 (as printed in the paper).

(a) $\tan\theta_c < \frac{3}{1+\sqrt{3}}$ (b) $\tan\theta_c > \frac{3}{1+\sqrt{3}}$ (c) $\sin\theta_c = \frac{3}{1+\sqrt{3}}$ (d) $\sin\theta_c > \frac{3}{1+\sqrt{3}}$

Solution

Correct option: A. **Method:** The ray meets the hypotenuse at an angle of incidence of 60°, so Snell's law gives sin r = sin 60° sin θ_c = (√3/2) sin θ_c, where r is the angle of refraction. Inside the prism the ray meets the vertical face at an incidence of 60° - r and the base at an incidence of 30° + r, and both must exceed θ_c. The binding condition, 60° - r > θ_c, gives tan θ_c < (3 - √3)/2, about 0.63, which is less than 3/(1 + √3), about 1.10, so only option (a) can hold. Options (c) and (d) are impossible, since sin θ_c cannot exceed 1.



Solution diagram: the ray path in the prism, with the angles marked.

Answer: (A).

Question 3 (Part A-1, single correct, +3 / -1)

A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} Nm/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of the magnetic field B (in Tesla) is approximately

(a) 10^{-2} (b) 10^{-3} (c) 10^{-1} (d) 10^{-4}

Solution

Correct option: B. **Method:** At equilibrium, NiAB = kθ. So B = kθ/(NiA) = $10^{-6} \times (\pi/180)/(10^{-3} \times 175 \times 10^{-4})$ = 9.97×10^{-4} T, about 10^{-3} T.

Answer: (B).

Question 4 (Part A-1, single correct, +3 / -1)

The figure shows an arrangement of charges [-2q at O (0, 0), +q at A (a, 0) and +q at B (-a, 0)]. The electric field E at point P (r, 0) can be expressed as (for r >> a) E = $(1/4\pi\varepsilon_0)(\alpha Q/r^n)$ $\hat{r}$ (where $Q = qa^2$). Choose the correct option.

Figure: on the x-axis, a charge +q at B (-a, 0), a charge -2q at O (0, 0), a charge +q at A (a, 0), and a point P (r, 0) to the right.

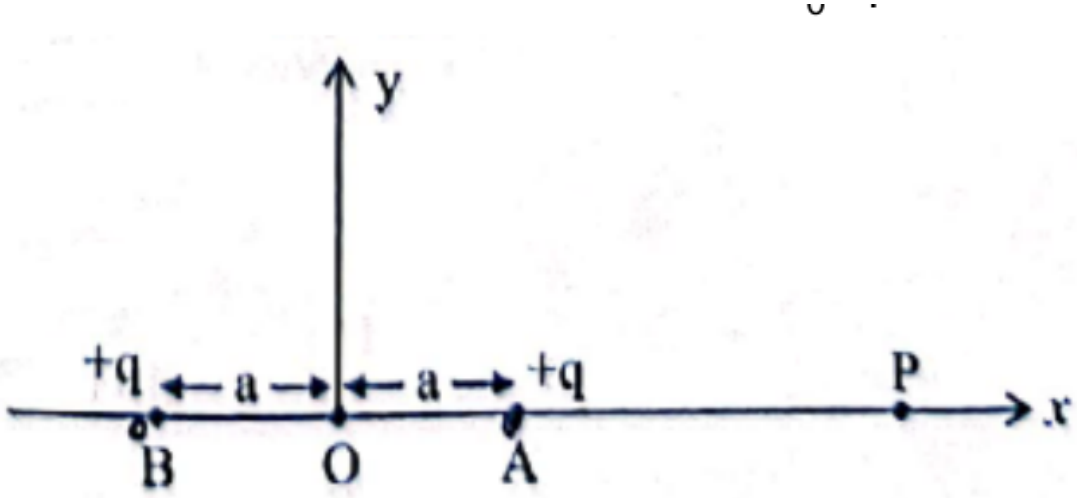


Figure for Question 4 (as printed in the paper).

(a) $\alpha = 3, n = 3$ (b) $\alpha = 3, n = 4$ (c) $\alpha = 6, n = 3$ (d) $\alpha = 6, n = 4$

Solution

Correct option: D. **Method:** $E = kq[1/(r - a)^2 - 2/r^2 + 1/(r + a)^2]\ \hat{r}$, with $k = 1/4\pi\varepsilon_0$. Combining the first and third terms gives $E = 2kq[3a^2r^2 - a^4]/[r^2(r^2 - a^2)^2]\ \hat{r}$. For $r >> a$ this is $E = 6kqa^2/r^4\ \hat{r} = (1/4\pi\varepsilon_0)(6Q/r^4)\ \hat{r}$. So $\alpha = 6$ and $n = 4$.

Answer: (D).

Question 5 (Part A-1, single correct, +3 / -1)

Charge Q coulomb is uniformly distributed throughout the volume of a solid hemisphere of radius R meter. The potential at the centre 'O' of the hemisphere in volts is Figure: a solid hemisphere of radius R with its centre marked O at the centre of the flat circular face.

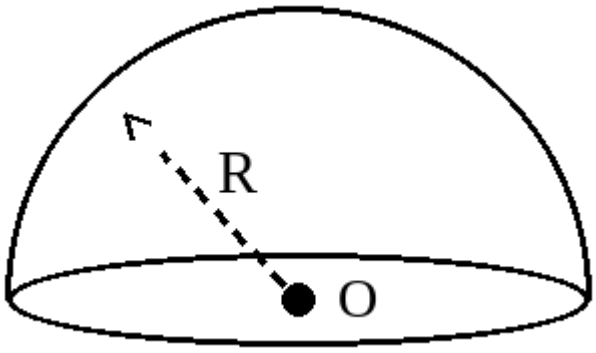


Figure for Question 5 (as printed in the paper).

(a) $(1/4\pi\varepsilon_0)(3Q/2R)$ (b) $(1/4\pi\varepsilon_0)(3Q/4R)$ (c) $(1/4\pi\varepsilon_0)(Q/4R)$ (d) $(1/4\pi\varepsilon_0)(Q/8R)$

Solution

Correct option: A. **Method:** For a uniformly charged solid sphere the potential at the centre is $(3/2)(1/4\pi\varepsilon_0)(Q/R)$. A hemisphere carries half the charge at the same radii, so by symmetry its potential at O is half of that: $V_0 = (1/4\pi\varepsilon_0)(3Q/2R)$.

Answer: (A).

Question 6 (Part A-1, single correct, +3 / -1)

The rear surface of an equi-concave lens of radius of curvature R and refractive index μ has been silvered; the lens behaves as a mirror. The focal length and the nature of the resulting mirror are Figure: an equi-concave (biconcave) lens with its rear surface silvered, shown shaded on the right side.

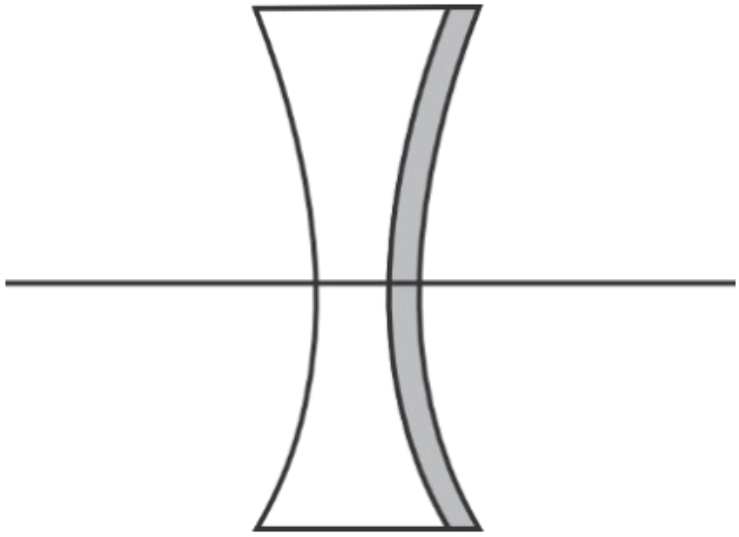


Figure for Question 6 (as printed in the paper).

(a) $\frac{R}{2(2\mu - 3)}$ converging (b) $\frac{R}{2(2\mu - 1)}$ converging (c) $\frac{R}{2(2\mu + 1)}$ diverging (d) $\frac{R}{2(2\mu - 1)}$ diverging

Solution

Correct option: D. **Method:** For a silvered lens, $1/F = 2/f_{lens} + 1/f_{mirror}$. Here $1/f_{lens} = (\mu - 1)(-2/R)$ and $f_{mirror} = -R/2$, so $1/F = -4(\mu - 1)/R - 2/R = -2(2\mu - 1)/R$. Hence $F = -R/[2(2\mu - 1)]$. The negative focal length means the mirror is diverging.

Answer: (D).

Question 7 (Part A-1, single correct, +3 / -1)

In Millikan's oil drop experiment, a charged drop falls vertically down with a terminal velocity v. If an electric field E is applied vertically upward, it moves with terminal velocity 2v in the upward direction. If the electric field reduces to E/2 then the terminal velocity of the drop becomes

(a) v (b) 2v (c) $\frac{3}{2}$ — v (d) $\frac{v}{2}$ —

Solution

Correct option: D. **Method:** While falling, the viscous drag balances the weight: F = mg. With the field E upward, qE = mg + 2F = 3mg (the drag is 2F, acting downward, since the drop moves up with speed 2v). With the field E/2, the upward electric force is qE/2 = 3mg/2, so the net upward force is mg/2. The drag at terminal velocity is then mg/2 = F/2, and since drag is proportional to speed, the terminal velocity is v/2 upward.

Answer: (D).

Question 8 (Part A-1, single correct, +3 / -1)

Seven equal positive charges of magnitude q each are placed at the seven corners of a cube of side l. The force experienced by a charge Q placed at the centre of the cube is (given $1/4\pi\varepsilon_0 = K$)

(a) Zero (b) $\frac{2KQq}{3l^2}$ (c) $\frac{4KQq}{3l^2}$ (d) $\frac{7KQq}{3l^2}$

Solution

Correct option: C. **Method:** The seven charges sit at seven corners, so six of them cancel in pairs by symmetry. The net force is the force from the single unpaired charge. The centre-to-corner distance is $(\sqrt{3}/2)l$, so $F = KqQ/[(3/4)l^2] = 4KQq/(3l^2)$.

Answer: (C).

Question 9 (Part A-1, single correct, +3 / -1)

The magnetic moment (p_m) and the moment of inertia (I) of a magnetic needle are $p_m = 8.7 \times 10^{-2}$ A.m^2 and $I = 11.5 \times 10^{-6}$ kg.m^2 respectively. The magnetic needle performs 10 complete oscillations in 6.70 s when suspended in a uniform horizontal magnetic field B. The magnitude of the magnetic field is

(a) 2.10 mT (b) 11.6 mT (c) 11.6 T (d) 1.5 T

Solution

Correct option: B. **Method:** The period is $T = 6.70/10 = 0.67$ s. For a needle, $T = 2\pi\sqrt{(I/MB)}$, so $B = 4\pi^2 I/(MT^2) = 4\pi^2 \times 11.5 \times 10^{-6}/(8.7 \times 10^{-2} \times 0.67^2) = 11.62 \times 10^{-3}$ T = 11.6 mT.

Answer: (B).

Question 10 (Part A-1, single correct, +3 / -1)

The resistance between the points A and B in the infinite circuit shown, in terms of R, is Figure: an infinite ladder network. The top rail has resistors R, 2R, 4R, 2^3R and so on in series, the bottom rail has the same, and shunt resistors R, 2R, 4R, 2^3R and so on join the two rails, continuing to infinity. Terminals A and B are at the left end.

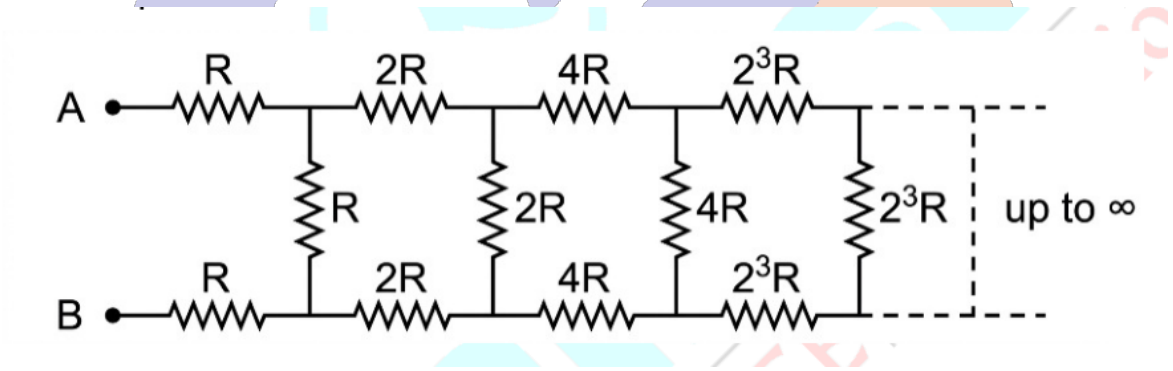


Figure for Question 10 (as printed in the paper).

(a) $\frac{5+\sqrt{41}}{4}$R (b) $\frac{5+\sqrt{41}}{4}$R (c) $\frac{5+\sqrt{41}}{2}$R (d) $(\sqrt{3}+1)$R

Solution

Correct option: A. **Method:** Let the resistance between A and B be x. The ladder is self-similar with a scale factor of 2, so after the first series resistors R + R = 2R, the remainder of the ladder has resistance 2x in parallel with the first shunt R. Hence $x = 2R + 2xR/(2x + R)$. This gives $2x^2 - 5Rx - 2R^2 = 0$, so $x = (5 + \sqrt{41})R/4$ (the positive root).

Answer: (A).

Question 11 (Part A-1, single correct, +3 / -1)

Work function of metal A is equal to the ionisation energy of the hydrogen atom in the first excited state. Work function of metal B is equal to the ionisation energy of singly-ionised helium atom in the second orbit. Photons of the same energy E are incident on the two metals A and B. Maximum kinetic energy of photoelectrons emitted from metal A is twice that of the photoelectrons emitted from metal B. The value of the photon energy E is

(a) 20.8 eV (b) 32.2 eV (c) 24.6 eV (d) 23.8 eV

Solution

Correct option: D. **Method:** $\Phi_A = 13.6/2^2 = 3.4$ eV (hydrogen, n = 2). $\Phi_B = 13.6 \times 2^2/2^2 = 13.6$ eV (He^+, Z = 2, n = 2). Einstein's equation gives E - 3.4 = 2(E - 13.6), so E = 27.2 - 3.4 = 23.8 eV.

Answer: (D).

Question 12 (Part A-1, single correct, +3 / -1)

During an excavation in the ancient volcanic caves, an archaeologist discovered a sample of a rare mineral trapped inside a sealed rock chamber. Laboratory analysis of the sample showed that the mineral originally contained a radioactive isotope X, which decays into a stable element Y. The radioactive isotope has a half-life $T = 10^4$ year. Due to contamination from underground water, 20 % of the stable element got washed away before the sample was analysed. At the time of the analysis, the ratio of remaining (undecayed) radioactive isotope X to stable element Y in the sample is found to be 1 : 5.6. Assuming that no radioactive material X was lost from the sample, the archaeologist claimed that the actual age of the rock is

(a) 1×10^4 year (b) $\sqrt{3} \times 10^4$ year (c) 3×10^4 year (d) $2\sqrt{3} \times 10^4$ year

Solution

Correct option: C. **Method:** Let N_0 be the initial number of X atoms and N the number remaining. The Y formed is N_0 - N, of which 20 % was washed away, so the Y present is $0.8(N_0 - N)$. Given $N/[0.8(N_0 - N)] = 1/5.6$, we get $5.6N = 0.8N_0 - 0.8N$, so $N = N_0/8 = N_0/2^3$. Three half-lives have passed, so $t = 3T = 3 \times 10^4$ year.

Answer: (C).

Question 13 (Part A-1, single correct, +3 / -1)

Two identical particles of mass m each are moving in circular orbits under a central potential given by $V(r) = \frac{K}{r^n}$, where K is a constant. If the radii of their orbits are r_1 and r_2 while the time periods are T_1 and T_2 respectively, the correct mathematical relation is

(a) $T_1^2 r_1^{-(n+2)} = T_2^2 r_2^{-(n+2)}$ (b) $T_1^2 r_1^{-3} = T_2^2 r_2^{-3}$

(c) $T_1^2 r_1^{-(2n-1)} = T_2^2 r_2^{-(2n-1)}$ (d) $T_1^2 r_1^{-(n+1)} = T_2^2 r_2^{-(n+1)}$

Solution

Correct option: A. **Method:** The force is $F = -dV/dr$, of magnitude nK/r^{n+1}. For a circular orbit, $mv^2/r = nK/r^{n+1}$, so v^2 is proportional to $1/r^n$ and v is proportional to $r^{-n/2}$. The period $T = 2\pi r/v$ is proportional to $r^{(n+2)/2}$, so T^2 is proportional to r^{n+2}, and $T^2 r^{-(n+2)}$ is the same for both orbits.

Answer: (A).

Question 14 (Part A-1, single correct, +3 / -1)

The figure represents a voltage regulator circuit using a zener diode. The breakdown voltage of the zener diode is 5.5 V and the load resistance is 5 kilo ohm. The series resistance R is 1 kilo ohm. If the battery voltage varies from 8 V to 14 V then what is the minimum and maximum value of current through the zener diode? Figure: a voltage regulator circuit. A battery feeds a series resistor R = 1.0 kΩ, after which a zener diode and the load resistor R_L = 5.0 kΩ are connected in parallel.

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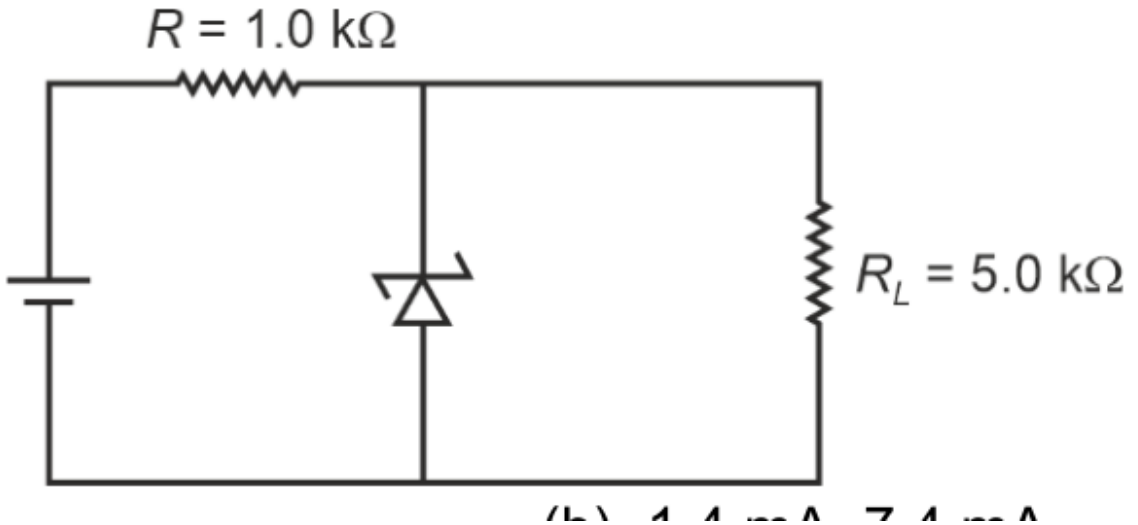


Figure for Question 14 (as printed in the paper).

(a) 2.5 mA, 8.5 mA (b) 1.4 mA, 7.4 mA (c) 1.1 mA, 1.1 mA (d) 0.5 mA, 1.5 mA

Solution

Correct option: B. **Method:** The zener holds the load voltage at 5.5 V, so the load current is fixed at 5.5/5 kΩ = 1.1 mA. The current through the series resistor is (V - 5.5)/1 kΩ: at 8 V it is 2.5 mA, so the zener current is 2.5 - 1.1 = 1.4 mA (minimum). At 14 V it is 8.5 mA, so the zener current is 8.5 - 1.1 = 7.4 mA (maximum).

Answer: (B).

Question 15 (Part A-1, single correct, +3 / -1)

Two coherent electromagnetic plane waves, of exactly equal amplitude, propagating with a very small angle of divergence ψ, fall almost normally on a distant screen to produce interference fringes with a separation (fringe width) x = 1.0 mm between neighbouring maxima. The wavelength of light is λ = 6000 Å. The angle of divergence ψ

(a) is estimated to be ψ = 6.0 milliradians (b) is estimated to be ψ = 0.6 milliradians

(c) is estimated to be ψ = 1.2 milliradians (d) cannot be estimated from this data

Solution

Correct option: B. **Method:** The fringe width is $x = \lambda/\psi$, so $\psi = \lambda/x = 6000 \times 10^{-10}/10^{-3} = 6 \times 10^{-4}$ rad = 0.6 milliradians.

Answer: (B).

Question 16 (Part A-1, single correct, +3 / -1)

In a line-of-sight radio communication, there is a distance of about 50 km between transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be (Radius of the Earth = 6400 km)

(a) 20 m (b) 51 m (c) 32 m (d) 40 m

Solution

Correct option: C. **Method:** The maximum line-of-sight distance is $d = \sqrt{(2R_e h_t)} + \sqrt{(2R_e h_R)}$. So $50000 = \sqrt{(2 \times 6400 \times 10^3 \times h_t)} + \sqrt{(2 \times 6400 \times 10^3 \times 70)}$. The second term is about 29933 m, leaving $\sqrt{(2 \times 6400 \times 10^3 \times h_t)} = 20067$, so h_t is about 31.5 m. The nearest option is 32 m.

Answer: (C).

Question 17 (Part A-1, single correct, +3 / -1)

In an arrangement, three polarisers P, Q and S have been set up in sequence such that transmission axis of S is crossed with respect to that of P. The transmission axis of Q is inclined at 60° to the transmission axis of S. When a beam of unpolarised light of intensity I_0 is incident on P, the intensity of light transmitted by the three polarisers in succession is I. The ratio $I : I_0$ is equal to

(a) 1 : 16 (b) 3 : 16 (c) 3 : 32 (d) 10 : 18

Solution

Correct option: C. **Method:** Unpolarised light through P gives $I_0/2$. Q is at 30° to P (since S is crossed with P and Q is 60° to S), so the intensity becomes $(I_0/2)\cos^2 30°$. S is at 60° to Q, so $I = (I_0/2)\cos^2 30° \cos^2 60° = (I_0/2) \times (3/4) \times (1/4) = 3I_0/32$. Hence $I : I_0 = 3 : 32$.

Answer: (C).

Question 18 (Part A-1, single correct, +3 / -1)

A solid sphere of mass m and radius R rolls without slipping on a horizontal surface. It collides with a light spring of stiffness constant K with kinetic energy E. If the surface under the spring is smooth (frictionless), then the maximum compression of the spring is

(a) $\sqrt{\frac{2E}{K}}$ (b) $\sqrt{\frac{10E}{7K}}$ (c) $\sqrt{\frac{E}{K}}$ (d) $\sqrt{\frac{7E}{10K}}$

Solution

Correct option: B. **Method:** The total kinetic energy is $E = (1/2)mv^2 + (1/2)(2/5\ mR^2)(v^2/R^2) = (7/10)mv^2$, so the translational part is $(1/2)mv^2 = 5E/7$. On the smooth surface there is no friction, so the rotation is unaffected and only the translational energy goes into the spring: $(1/2)Kx^2 = 5E/7$, so $x = \sqrt{(10E/7K)}$.

Answer: (B).

Question 19 (Part A-1, single correct, +3 / -1)

When a mass of 9.0 kg is suspended vertically from the free end of the wire of a sonometer, it resonates with a given tuning fork forming standing waves with five antinodes between the two bridges. When the 9.0 kg mass is replaced by another unknown mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The mass M is

(a) 5.4 kg (b) 15 kg (c) 25 kg (d) 35 kg

Solution

Correct option: C. **Method:** The frequency is $f = (n/2l)\sqrt{(T/\mu)}$. For the same fork and the same length, $5\sqrt{(9g)} = 3\sqrt{(Mg)}$. Squaring gives $25 \times 9 = 9M$, so M = 25 kg.

Answer: (C).

Question 20 (Part A-1, single correct, +3 / -1)

A thin rectangular metal plate of surface area 0.1 m^2 is placed on a large horizontal table. There are a few drops of a liquid between the metal plate and the table forming a thin liquid layer of 0.3 mm thickness. The plate is pulled horizontally with the help of a string passing over a massless and frictionless fixed pulley. When a mass of 10 g is attached to the vertically hanging free end of the string, the plate is observed to move gently towards the pulley with a constant speed of 8.5 cms^{-1}. The coefficient of viscosity of the liquid is approximately

(a) 3.5 Pascal-s (b) 0.035 Pascal-s (c) 0.0035 Pascal-s (d) 0.0025 Pascal-s

Solution

Correct option: C. **Method:** The plate moves at constant speed, so the pulling force equals the viscous force: $(0.010\ kg)g = \eta A(dv/dz)$. Hence $0.098 = \eta \times 0.1 \times (0.085/0.0003)$, giving $\eta = 3.53 \times 10^{-3}$ Pa-s, about 0.0035 Pa-s.

Answer: (C).

Question 21 (Part A-1, single correct, +3 / -1)

In an experiment for determination of the focal length of a thin convex lens, the object distance is measured as 15 cm with 1% error and the image distance is measured as 30 cm with 2% error. From the given data, the focal length of the lens is estimated with n% error. The value of n is

(a) 1.33 (b) 3 (c) 0 (d) 2

Solution

Correct option: A. **Method:** From $1/f = 1/u + 1/v$, f = 10 cm. Differentiating, $\Delta f/f^2 = \Delta u/u^2 + \Delta v/v^2$, so $\Delta f/f = f(\Delta u/u^2 + \Delta v/v^2) = 10 \times (0.15/225 + 0.6/900) = 10 \times (1/1500 + 1/1500) = 4/3$ %. Hence n = 1.33.

Answer: (A).

Question 22 (Part A-1, single correct, +3 / -1)

The Earth is revolving around the Sun in a circular orbit with a period of revolution equal to 1 year. Suppose that during its motion around the Sun, the mass of the Sun doubles suddenly, the time period of revolution of the Earth will then become

(a) $\frac{1}{2}$ year (b) $\frac{2\sqrt{2}}{3\sqrt{3}}$ year (c) $\frac{2}{3\sqrt{3}}$ year (d) $\frac{1}{2\sqrt{2}}$ year

Solution

Correct option: C. **Method:** Just after the mass doubles, the Earth's speed is unchanged, $v_0^2 = GM/R$, but the energy becomes $E = (1/2)mv_0^2 - G(2M)m/R = -3GMm/(2R)$. For the new orbit, $E = -G(2M)m/(2a)$, so $a = 2R/3$. The new period is $T' = 2\pi\sqrt{(a^3/(2GM))} = 2\pi\sqrt{((8R^3/27)/(2GM))} = (2/(3\sqrt{3})) \times 2\pi\sqrt{(R^3/GM)} = 2/(3\sqrt{3})$ year.

Answer: (C).

Question 23 (Part A-1, single correct, +3 / -1)

Three metallic wires form an isosceles right-angled triangle AOB with wire AB parallel to x-axis and the vertex O at the origin. The length of the wire OA = length OB = $2\sqrt{2}$ m. A current of 10 A flows in the loop along OABO. A magnetic field $B = 2.5(\hat{i} - \hat{k})$ T exists in the region $y \geq 1$. The force acting on the triangle due to this field is Figure: an isosceles right-angled triangle AOB with the vertex O at the origin and the right angle at O. The wire AB is horizontal at the top, and current flows along O to A to B to O, as shown by arrows.

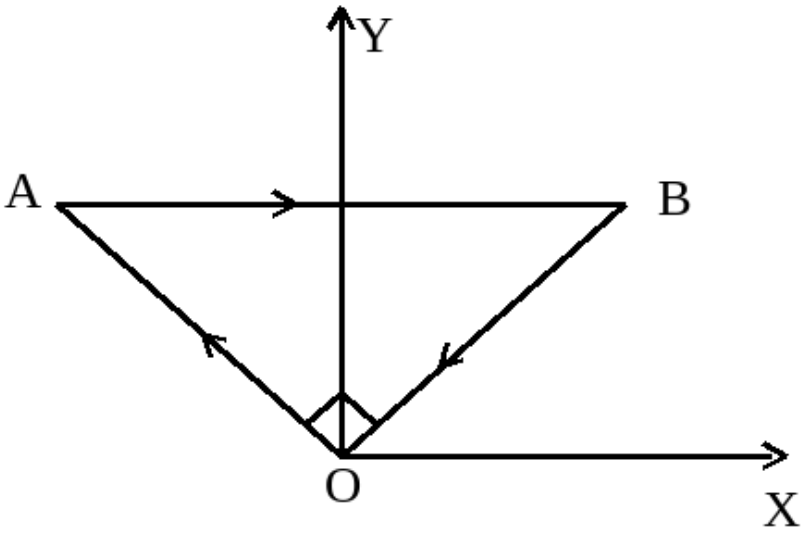


Figure for Question 23 (as printed in the paper).

(a) 50ĵN (b) 50(î + k̂)N (c) 50(ĵ + î)N (d) Zero

Solution

Correct option: A. **Method:** The vertices are O(0, 0), A(-2, 2) and B(2, 2), since OA and OB are at 45° to the axes. The field exists only for y ≥ 1, so the part of the loop inside the field is D-A-B-C, where C and D are the points where OA and OB cross y = 1. The chord CD is 2 m long and horizontal. The force on the closed loop C-D-A-B-C in a uniform field would be zero, so the force on D-A-B-C is the negative of the force on the chord CD: F = -I(CD vector) × B = -10 × (-2î) × 2.5(î - k̂) = 50ĵ N.

Answer: (A).

Question 24 (Part A-1, single correct, +3 / -1)

A spring is in a compressed state under the action of a constant force P. Compression of the spring is d. Suppose direction of the force is reversed suddenly with its magnitude changed to 2P; the maximum extension of the spring will now be Figure: a spring fixed to a wall on the left. The upper diagram shows the spring at its natural length; the lower diagram shows it compressed by a distance d under a constant force P acting to the left.

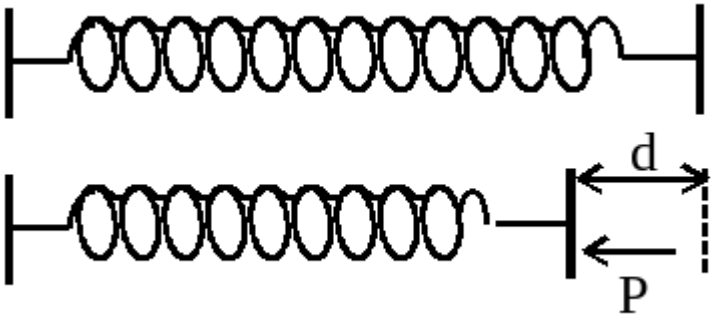


Figure for Question 24 (as printed in the paper).

(a) 2d (b) 3d (c) 4d (d) 5d

Solution

Correct option: D. **Method:** From P = Kd, the spring constant is K = P/d. Let the maximum extension be x_m. By work-energy from the compressed position (-d) to x_m, the change in kinetic energy is zero: $2P(x_m + d) - (1/2)K(x_m^2 - d^2) = 0$. This gives $2Kd(x_m + d) = (1/2)K(x_m - d)(x_m + d)$, so $4d = x_m - d$ and $x_m = 5d$.

Answer: (D).

Question 25 (Part A-1, single correct, +3 / -1)

In a process, one mole of a monoatomic gas is heated from initial temperature 27°C, in such a way that its molar heat capacity is constant equal to 2R. During the process, volume of the gas is doubled. The final temperature of the gas becomes

(a) 108°C (b) 927°C (c) 527°C (d) 2127°C

Solution

Correct option: B. **Method:** For a polytropic process, $C = C_V + R/(1 - x)$. With C = 2R and $C_V = 3R/2$, we get $2R = 3R/2 + R/(1 - x)$, so x = -1 and PV^{-1} = constant. Hence TV^{-2} = constant, so $T_2 = T_1(V_2/V_1)^2 = 300 \times 4 = 1200$ K = 927°C.

Answer: (B).

Question 26 (Part A-1, single correct, +3 / -1)

When placed in a liquid of density d_1, a solid sphere sinks displacing a volume V of the liquid. When placed in a second liquid of density d_2, the same sphere floats with half of its volume appearing over the surface of the liquid. When placed in a third liquid of density d_3, the same sphere floats with $\frac{1}{4}$th of its volume over the surface of the liquid. The buoyant force exerted by the liquid of density d_3 on the sphere is

(a) Vd_3g (b) $\frac{1}{2}Vd_1g$ (c) $\frac{1}{2}Vd_2g$ (d) $\frac{1}{4}Vd_3g$

Solution

Correct option: C. **Method:** The sphere sinks fully in liquid 1, so its volume is V. In liquid 2 it floats with half its volume submerged, so its weight is $mg = (V/2)d_2g$. In liquid 3 it also floats, so the buoyant force equals its weight: $F = mg = (1/2)Vd_2g$.

Answer: (C).

Question 27 (Part A-1, single correct, +3 / -1)

A flat rectangular l × b loop and a long straight wire lie in the same plane (see figure). Current in the wire varies according to $I = I_0 \sin\omega t$. The amplitude of emf induced in the loop is (Given b = 2a) Figure: a long straight vertical wire on the left carrying current I upward, and a flat rectangular loop of length l and breadth b to its right. The near edge of the loop is at a distance a from the wire.

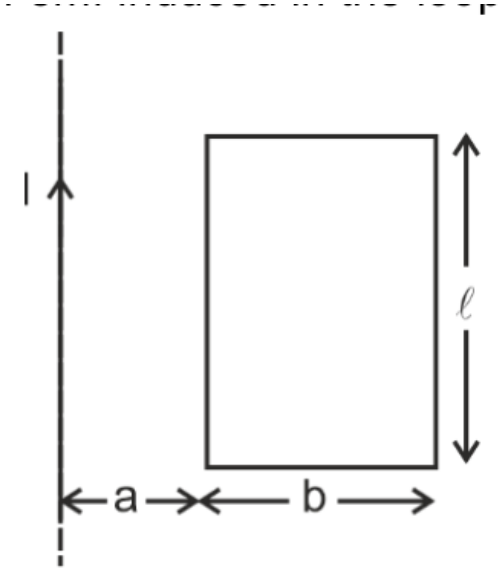


Figure for Question 27 (as printed in the paper).

(a) $\frac{\mu_0 \omega I_0 l}{2\pi}$ ln 3 (b) $\frac{\mu_0 \omega I_0 l}{2\pi}$ ln 2 (c) $\frac{3\mu_0 \omega I_0 l}{2\pi}$ (d) $\frac{\mu_0 \omega I_0 l}{\pi}$

Solution

Correct option: A. **Method:** The flux through the loop is $\varphi = \int_a^{a+b} (\mu_0 I/2\pi x)\, l\, dx = (\mu_0 Il/2\pi) \ln((a + b)/a)$. With $b = 2a$, $(a + b)/a = 3$, so $\varphi = (\mu_0 I_0 l/2\pi) \ln 3 \times \sin\omega t$. The induced emf is $-d\varphi/dt$, whose amplitude is $(\mu_0 \omega I_0 l/2\pi) \ln 3$.

Answer: (A).

Question 28 (Part A-1, single correct, +3 / -1)

There are two concentric and coplanar coils of radii R_1 and R_2 ($R_1 >> R_2$), and number of turns N_1 and N_2 respectively. The current in the second (smaller) coil is changing at a constant rate $dI_2/dt = k$. The magnitude of induced emf in the first (larger) coil will be Figure: two concentric coplanar circular coils, the inner coil of radius R_2 and the outer coil of radius R_1.

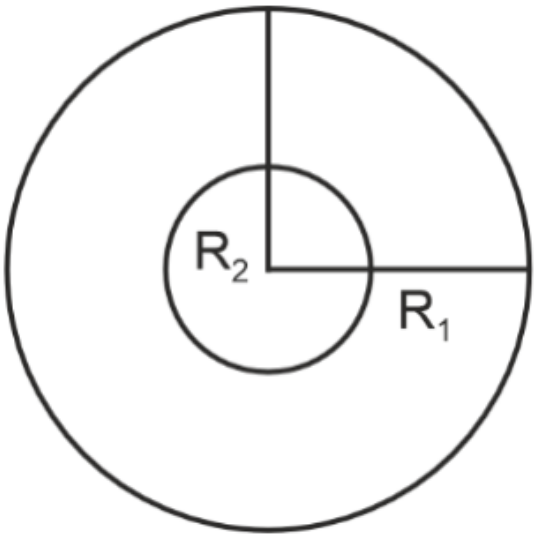


Figure for Question 28 (as printed in the paper).

(a) $\frac{\mu_0 \pi N_1 N_2}{2} R_1 R_2 k$

(b) $\mu_0 \pi N_1^2 N_2^2 \frac{R_2}{2R_1} k$

(c) $\mu_0 \pi N_1 N_2^2 \frac{R_1}{2R_2} k$

(d) $\mu_0 \pi N_1 N_2^2 \frac{R_2}{2R_1} k$

Solution

Correct option: D. **Method:** The mutual inductance is found by passing a current I through the outer coil: the field at the centre is $\mu_0 I N_1/(2R_1)$, and the flux through the inner coil is $M = \mu_0 \pi N_1 N_2 R_2^2/(2R_1)$. The induced emf in the outer coil is $M(dI_2/dt) = \mu_0 \pi N_1 N_2 R_2^2 k/(2R_1)$.

Answer: (D).

Question 29 (Part A-1, single correct, +3 / -1)

A beam of light in air is incident at an angle of incidence i = 75° on a large block made of three parallel layers of different transparent materials stacked on top of each other, as shown. The thickness of each layer is t = 10.0 cm. Refractive indices of the three successive layers with respect to air are $n_1 = 2.40$, $n_2 = 2.00$ and $n_3 = 1.50$ respectively. The beam of light emerges from the third medium on the X axis at a distance ξ from O. The value of ξ is Figure: three horizontal layers stacked on top of each other, each of thickness t: $n_1 = 2.40$ on top, $n_2 = 2.00$ in the middle, $n_3 = 1.50$ at the bottom, with air above and below. A ray enters from the air at an angle of incidence i = 75° to the vertical, and the X axis runs along the bottom surface of the third layer with its origin O below the entry point.

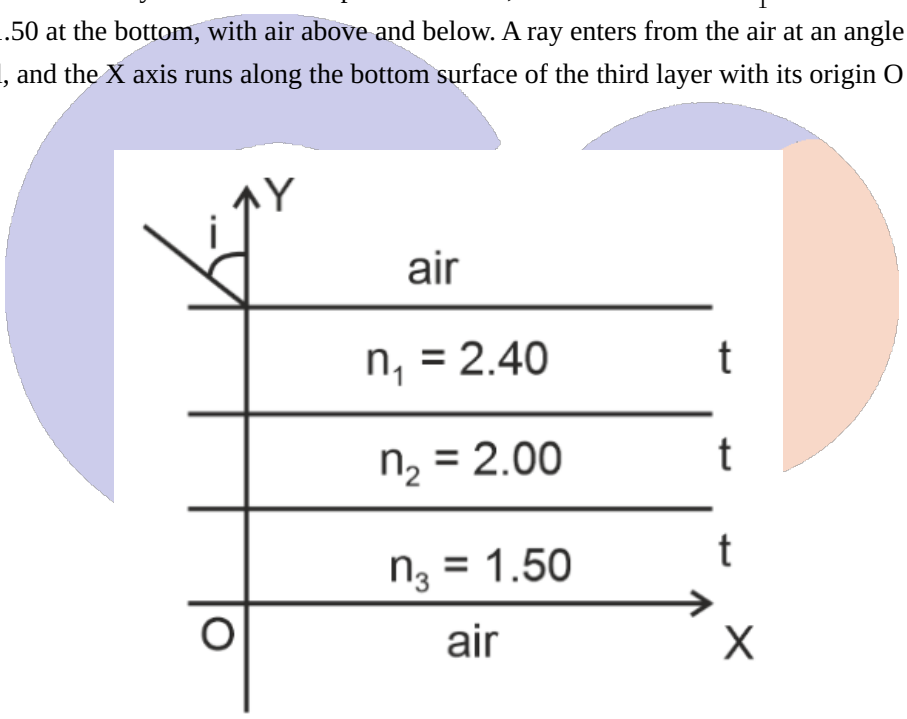


Figure for Question 29 (as printed in the paper).

(a) 30.3 cm (b) 23.4 cm (c) 18.3 cm (d) 9.02 cm

Solution

Correct option: C. **Method:** By Snell's law, $\sin 75° = 2.4 \sin\theta_1 = 2 \sin\theta_2 = 1.5 \sin\theta_3$, giving $\theta_1 = 23.7°$, $\theta_2 = 28.9°$ and $\theta_3 = 40.0°$. The sideways shift in each layer is t tan θ, so $\xi = 10(\tan 23.7° + \tan 28.9° + \tan 40.0°) = 10 \times 1.83 = 18.3$ cm.

Answer: (C).

Question 30 (Part A-1, single correct, +3 / -1)

A uniform rod AB of weight W is symmetrically suspended with the help of two identical un-stretchable light strings (1) and (2) of equal length from a rigid horizontal support. The rod is horizontal. The tension in the string

(1), immediately after the string (2) is burnt, would be Figure: a uniform rod AB suspended horizontally by two identical vertical strings (1) on the left and (2) on the right, hanging from a rigid horizontal support.

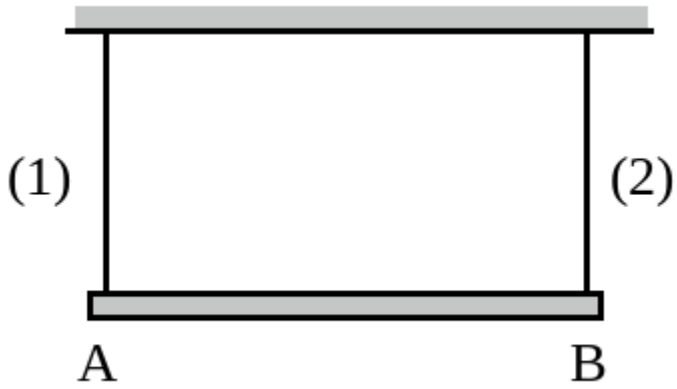


Figure for Question 30 (as printed in the paper).

(a) W (b) $\frac{3W}{4}$ (c) $\frac{W}{2}$ (d) $\frac{W}{4}$

Solution

Correct option: D. **Method:** Let the rod have length l and the tension in string (1) be T. Taking torque about the point where string (1) is attached, $mg(l/2) = (ml^2/3)\alpha$, so $\alpha = 3g/(2l)$. For the centre of mass, $mg - T = m\alpha(l/2) = 3mg/4$. Hence $T = mg/4 = W/4$.

Answer: (D).

Question 31 (Part A-1, single correct, +3 / -1)

Speed of sound is measured in a confined gas and found to be 0.632 times the RMS velocity (V_{rms}) of the gas molecules as determined from the temperature. The degree of freedom f and molar heat capacity C_V at constant volume of the gas are

(a) f = 10, $C_V = 5R$ (b) f = 6, $C_V = 3R$ (c) f = 5, $C_V \frac{5R}{2}$ (d) f = 3, $C_V \frac{3R}{2}$

Solution

Correct option: A. **Method:** The speed of sound is $\sqrt{(\gamma RT/M)}$ and $V_{rms} = \sqrt{(3RT/M)}$. So $\gamma = (0.632)^2 \times 3 = 1.2$. With $\gamma = (f + 2)/f$, we get $f + 2 = 1.2f$, so f = 10, and $C_V = fR/2 = 5R$.

Answer: (A).

Question 32 (Part A-1, single correct, +3 / -1)

In a wide container, water is always filled up to a height h. There is a nozzle (hole) of radius r_0 at its bottom. For some distance below the nozzle, water falls in a laminar stream. Radius r of stream as a function of y below the nozzle is expressed as [ignore the viscosity of water] Figure: a wide container kept filled with water up to a height h, with a nozzle (hole) of radius r_0 at its bottom. The water stream falls a distance y below the nozzle.

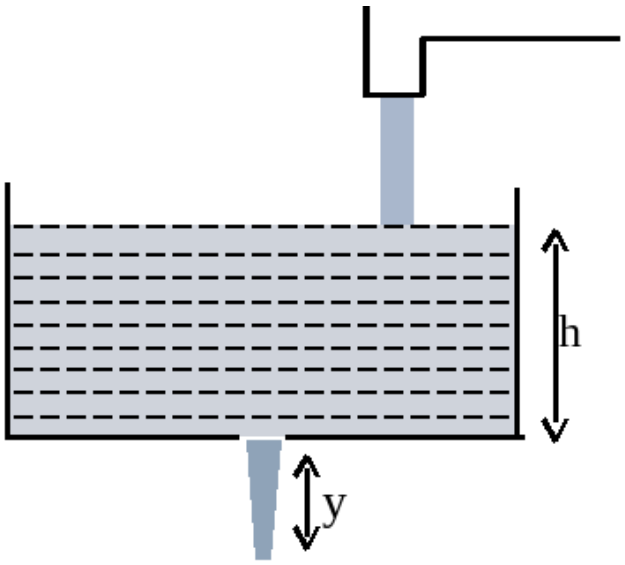


Figure for Question 32 (as printed in the paper).

(a) $r = \frac{r_0}{(1 + y/h)^{1/2}}$

(b) $r = \frac{r_0}{(1 + y/h)^{1/4}}$

(c) $r = r_0(1 + \frac{y}{h})^{1/2}$

(d) $r = \frac{r_0}{(1 + h/y)^{1/4}}$

Solution

Correct option: B. **Method:** By continuity, $\pi r_0^2 v_0 = \pi r^2 v$. Torricelli's law gives $v_0 = \sqrt{(2gh)}$ at the nozzle and $v = \sqrt{(2g(h + y))}$ at a depth y below it. Hence $r^2 = r_0^2 h/(h + y)$, so $r = r_0/(1 + y/h)^{1/4}$.

Answer: (B).

Question 33 (Part A-1, single correct, +3 / -1)

A point object of mass m starts moving from rest along x axis from the origin, under the action of a force which delivers a constant power P. The position x of the object at time t is

(a) $x = \left(\frac{8Pt^3}{9m}\right)^{1/2}$

(b) $x = \left(\frac{9Pt^3}{8m}\right)^{1/2}$

(c) $x = \left(\frac{8Pt^4}{27m}\right)^{1/3}$

(d) $x = \frac{2}{3}\left(\frac{Pt^3}{m}\right)^{1/2}$

Solution

Correct option: A. **Method:** Constant power means $Fv = P$, so $v(dv/dt) = P/m$, and integrating gives $v^2/2 = Pt/m$, so $v = \sqrt{(2Pt/m)}$. Integrating once more, $x = \int v\,dt = (2/3)\sqrt{(2P/m)}\, t^{3/2} = (8Pt^3/9m)^{1/2}$.

Answer: (A).

Question 34 (Part A-1, single correct, +3 / -1)

A black body, maintained at absolute temperature T, is emitting radiations of wavelength λ with maximum (peak) intensity. The emitted photons are capable of exciting the electron in a Bohr-model hydrogen atom from ground state (E_g = -13.60 eV) to the third excited state. Using Wien's constant $b = 2.90 \times 10^{-3}$ mK, the estimated value of temperature T is nearly

(a) 2.98×10^3 K

(b) 2.98×10^4 K

(c) 2.83×10^4 K

(d) 2.68×10^4 K

Solution

Correct option: B. **Method:** The third excited state is n = 4, so $\Delta E = 13.6(1 - 1/16) = 15 \times 13.6/16 = 12.75$ eV. The peak wavelength is $\lambda = hc/\Delta E = 1240 \times 16/(15 \times 13.6) = 97.25$ nm. By Wien's law, $T = b/\lambda = 2.9 \times 10^{-3}/(97.25 \times 10^{-9}) = 29820$ K, about 2.98×10^4 K.

Answer: (B).

Question 35 (Part A-1, single correct, +3 / -1)

In order to form standing waves in a medium, the following two waves are superposed: $y_1 = A \sin \pi(5x + 100t + 0.8)$ and $y_2 = A \sin \pi(5x - 100t)$. The variables x and y are in meter and t in second. The correct statement is that the

(a) Positions of first two displacement nodes (for x > 0) are x = 2 cm and x = 22 cm respectively

(b) Velocity amplitude is maximum at both the positions (for x > 0) x = 12 cm and x = 22 cm

(c) Pressure variation is maximum at both the positions (for x > 0) x = 22 cm and x = 32 cm

(d) Positions of first two displacement antinodes (for x > 0) are x = 2 cm and x = 22 cm respectively

Solution

Correct option: D. **Method:** Adding the waves gives $y = 2A \sin(5\pi x + 0.4\pi) \cos(100\pi t + 0.4\pi)$. Displacement nodes need $5\pi x + 0.4\pi = n\pi$, giving x = 12 cm and 32 cm for the first two. Displacement antinodes need $5\pi x + 0.4\pi = (2n - 1)\pi/2$, giving x = 2 cm and 22 cm for the first two. So option (d) is correct and (a) is wrong.

Answer: (D).

Question 36 (Part A-1, single correct, +3 / -1)

The density ρ of a solid sphere, of mass M and radius R, varies radially according to $\rho = \rho_0 \frac{1 - r}{2R}$, where ρ_0 is a constant. The moment of inertia of this sphere about a diameter is $I = \alpha MR^2$ with a equal to

(a) $\frac{28}{125}$ (b) $\frac{28}{75}$ (c) $\frac{7}{25}$ (d) $\frac{7}{50}$

Solution

Correct option: B. **Method:** The mass is $M = \int_0^R \rho_0(1 - r/2R)4\pi r^2\, dr = 4\pi\rho_0 \times 5R^3/24$. The moment of inertia about a diameter is $I = (2/3)\int_0^R \rho_0(1 - r/2R)4\pi r^4\, dr = (8\pi\rho_0/3) \times 7R^5/60 = 14\pi\rho_0 R^5/45$. Hence $\alpha = I/(MR^2) = (14/45)/(5/6) = 28/75$.

Answer: (B).

Question 37 (Part A-1, single correct, +3 / -1)

A concave mirror of radius of curvature R = 1.00 m is placed on the floor as shown. A small, light-weight ball falls from a height h = 1.20 m vertically along the principal axis towards the mirror. If the ball loses 16% of its energy after each collision with the mirror, the distance between the ball and its image formed by the mirror, when the ball reaches its maximum height after the 2nd collision, is nearly Figure: a concave mirror of radius of curvature R = 1.00 m placed on the floor like a bowl, with a small ball above it at a height h on the principal axis (shown by a dashed vertical line).

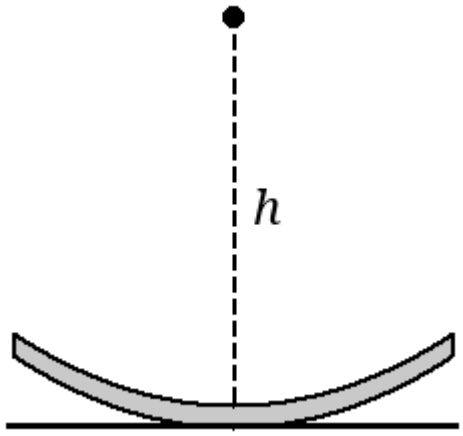


Figure for Question 37 (as printed in the paper).

(a) 0.75 m (b) 0.66 m (c) 0.55 m (d) 0.37 m

Solution

Method: After two collisions the ball rises to H' = 0.84 × 0.84 × 1.2 = 0.847 m. The mirror has focal length f = R/2 = 0.50 m. With the object at u = 0.847 m, the mirror formula gives 1/v = 1/f - 1/u = 2 - 1/0.847, so v = 1.22 m. The distance between the ball and its image is v - u = 1.22 - 0.85 = 0.37 m, option (d). **Note:** Aakash's answer key says (b) 0.66 m, but its own solution works out 0.37 m, option (d), and an independent solve also gives 0.37 m, so no key is claimed for this card.

Answer: disputed - the Aakash key (b) 0.66 m and its own solution (d) 0.37 m disagree. See the Note in the method.

Question 38 (Part A-1, single correct, +3 / -1)

The magnetic field vector in a plane electromagnetic wave is given by $B = 2 \times 10^{-7} \sin(kx + \omega t)\hat{j}$ T. It is given that speed of wave is $c = 3.00 \times 10^{8}$ ms^{-1} and $\omega = 1.50 \times 10^{11}$ s^{-1}

(a) The wavelength of the wave is 2 mm

(b) The frequency of the wave is nearly 2.39 GHz

(c) The electric field of the wave is $E = 60 \sin(kx - \omega t)\hat{k}$ Vm^{-1}

(d) The electric field of the wave is $E = 60 \sin(\omega t + kx)\hat{k}$ Vm^{-1}

Solution

Correct option: D. **Method:** $k = \omega/c = 1.5 \times 10^{11}/(3 \times 10^{8}) = 500$ m^{-1}. The frequency is $f = \omega/2\pi = 23.9$ GHz, not 2.39 GHz, so (b) is wrong. The wavelength is $\lambda = c/f = 12.6$ mm, not 2 mm, so (a) is wrong. $E_0 = B_0 c = 2 \times 10^{-7} \times 3 \times 10^{8} = 60$ Vm^{-1}, and E has the same phase $(kx + \omega t)$ as B and points along $\hat{k}$, so (d) is correct.

Answer: (D).

Question 39 (Part A-1, single correct, +3 / -1)

An astronomical telescope is used to view an object of size 4 cm kept at a distance of 20 m. The focal lengths of the objective and the eyepiece are 80 cm and 5 cm respectively. The final image is 25 cm from the eyepiece (with the eye close to the eyepiece). Statement 1: The size of the final image is 1.0 cm. Statement 2: The angular magnification is numerically 20

(a) Both statements 1 and 2 are wrong

(b) Statement 1 is correct but statement 2 is wrong

(c) Statement 1 is wrong but statement 2 is correct

(d) Both statements 1 and 2 are correct

Solution

Correct option: D. **Method:** For the objective, $1/v_e - 1/(-2000) = 1/80$, so $v_e = 250/3$ cm and the objective magnification is $m_o = (250/3)/2000 = 1/24$. For the eyepiece, the magnification is $m_e = 1 + D/f_e = 1 + 25/5 = 6$. The final image size is $4 \times (1/24) \times 6 = 1.0$ cm, so statement 1 is correct. The angular magnification is $(1/25)/(4/2000) = 20$, so statement 2 is correct.

Answer: (D).

Question 40 (Part A-1, single correct, +3 / -1)

A thin metal plate of infinite length extending in positive y direction carries a current of current density J all along its length as shown. The very small thickness of the plate is t. The breadth (much larger than thickness) of the plate lies exactly along the x axis. The magnetic field produced at a small distance 'a' above and very close to the surface of the plate is Figure: a thin metal plate of infinite length extending in the positive y direction, of breadth along the x axis and very small thickness t. The current density J flows along the plate in the +y direction. Coordinate axes x, y and z are shown.

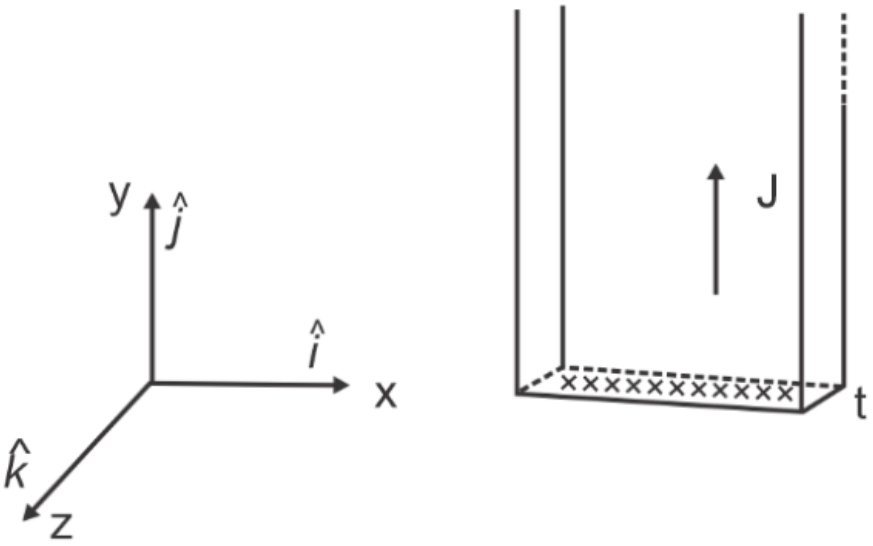


Figure for Question 40 (as printed in the paper).

(a) $\frac{\mu_0 Jt}{2}\hat{\imath}$ (b) $\frac{\mu_0 Jt}{2}\hat{\imath}$ (c) $\frac{\mu_0 Jt}{2}\hat{k}$ (d) $\frac{\mu_0 Jt}{a}\hat{\imath}$

Solution

Correct option: B. **Method:** Take a rectangular Amperian loop straddling the plate, of length l along x. The field is parallel to the plate on both sides, so $2Bl = \mu_0 I_{enc} = \mu_0 Jtl$, giving $B = \mu_0 Jt/2$. By the right-hand rule, with current along +y, the field just above the plate points along +x. So $B = +(\mu_0 Jt/2)\hat{\imath}$.

Answer: (B).

Question 41 (Part A-1, single correct, +3 / -1)

A uniform rod of mass M and length L is hinged freely on a horizontal peg at a variable distance ξ from one of its ends, so as to execute SHM with small angular amplitude in a vertical plane about the horizontal axis with time period T. If the minimum value of its time period $T_{min} = 1.0$ second, the approximate value of L is

(a) 24.8 cm (b) 43.0 cm (c) 56.8 cm (d) 99.2 cm

Solution

Correct option: B. **Method:** About the peg, the period is $T = 2\pi\sqrt{((l^2/12 + \xi^2)/(g\xi))}$. The quantity $l^2/(12\xi) + \xi$ is minimum when $\xi = l/\sqrt{12}$, and its minimum value is $l/\sqrt{3}$. So $T_{min} = 2\pi\sqrt{(l/(g\sqrt{3}))}$. With $T_{min} = 1.0$ s, $l = 0.438$ m = 43.8 cm, so the closest option is (b) 43.0 cm.

Answer: (B).

Question 42 (Part A-1, single correct, +3 / -1)

P and Q are two smooth weightless pulleys attached to the ceiling as shown. An un-stretchable, light string passes over both the pulleys. A weight W = 40 kg, resting on the floor, is tied to one end of the string. A ball of mass 20 kg is attached, at the other end A. The string PA is deflected by angle θ_0 when the ball A is released from rest. The ball at the end A is capable of just lifting up the weight W for the value of θ_0 Figure: two smooth weightless pulleys P (left) and Q (right) attached to the ceiling. A light string passes over both pulleys. A ball A hangs from the string at the left end, deflected by an angle θ_0 from the vertical (shown dashed), and a weight W rests on the floor at the right end, tied to the string over pulley Q.

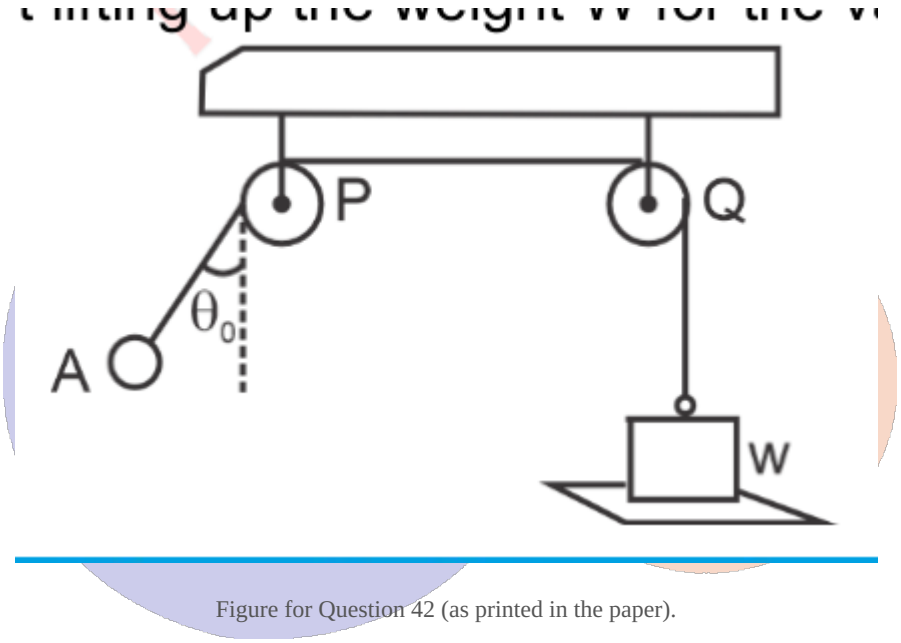


Figure for Question 42 (as printed in the paper).

(a) = 30° (b) = 45° (c) ≤ 53° (d) ≥ 60°

Solution

Correct option: D. **Method:** Released from rest at angle θ_0, the ball reaches the bottom with speed $v = \sqrt{(2gl(1 - \cos\theta_0))}$. At the bottom, $T - mg = mv^2/l$, so $T = 200 + 20 \times 2g(1 - \cos\theta_0)$. To lift the 40 kg weight, T must reach 400 N: $400 - 200 = 400(1 - \cos\theta_0)$, so $\cos\theta_0 = 1/2$ and $\theta_0 = 60°$. The ball can lift W when $\theta_0 \geq 60°$.

Answer: (D).

Question 43 (Part A-1, single correct, +3 / -1)

The position vector r of a particle of mass m moving in XY plane is given as a function of time by $r = R(\omega t - \sin\omega t)\hat{i} + R(1 - \cos\omega t)\hat{j}$ where R and ω are positive constants. The particle is momentarily at rest when

(a) $t = \frac{n\pi}{\omega}$ where n = 1, 3, 5... (b) y component of r is = $2R\,\hat{j}$

(c) the acceleration $a = \omega^2 R\,\hat{j}$ (d) the acceleration $a = -\omega^2 R\,\hat{j}$

Solution

Correct option: C. **Method:** The velocity is $v = R\omega(1 - \cos\omega t)\hat{i} + R\omega \sin\omega t\,\hat{j}$. It is zero only when $\sin\omega t = 0$ and $\cos\omega t = 1$, that is, $\omega t = 2n\pi$, so $t = 2n\pi/\omega$. Option (a) gives $\omega t = \pi, 3\pi, \ldots$ where v is not zero, so (a) is wrong. At those instants $y = R(1 - \cos\omega t) = 0$, so (b) is wrong. The acceleration is $a = R\omega^2 \sin\omega t\,\hat{i} + R\omega^2 \cos\omega t\,\hat{j} = R\omega^2\,\hat{j}$ at those instants, so (c) is correct.

Answer: (C).

Question 44 (Part A-1, single correct, +3 / -1)

A thin circular disk of mass m and radius R can rotate freely in a vertical plane about a pivot P on its rim (as in the figure). It is released when its centre is at C_1 at the level of the pivot in a vertical plane. The speed of the lowest point of the disk when the centre is vertically below the pivot P at C_2 is Figure: a thin circular disk pivoted at a point P on its rim. The dashed circle shows the disk with its centre at C_1, level with the pivot P; the solid circle shows the disk with its centre at C_2, vertically below P.

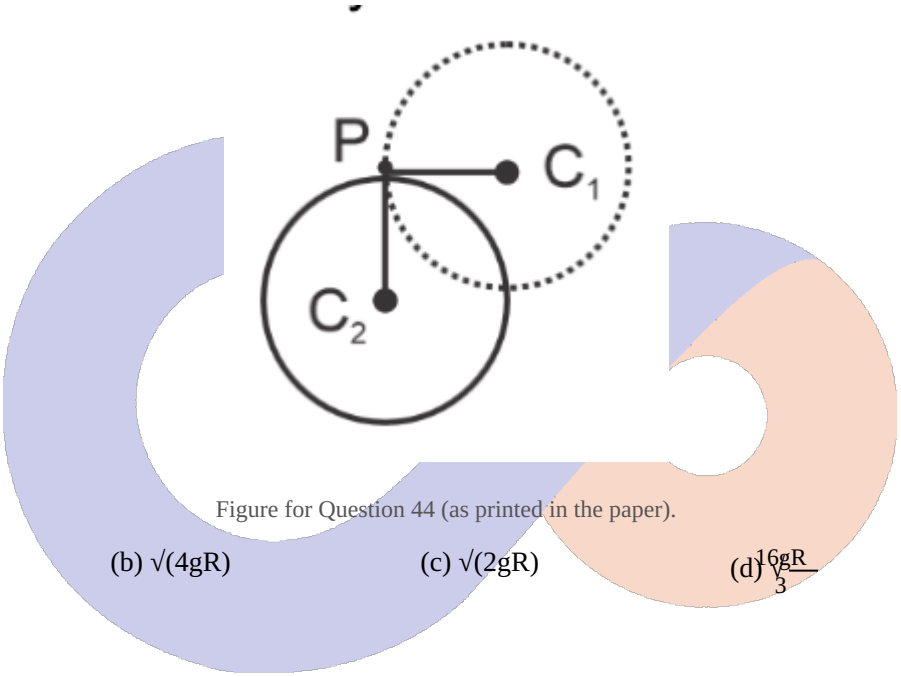


Figure for Question 44 (as printed in the paper).

(a) $\sqrt{\frac{4gR}{3}}$ (b) $\sqrt{(4gR)}$ (c) $\sqrt{(2gR)}$ (d) $\sqrt{\frac{16gR}{3}}$

Solution

Correct option: D. **Method:** The moment of inertia about the pivot is $I_P = (1/2)mR^2 + mR^2 = (3/2)mR^2$. By conservation of energy, $mgR = (1/2)I_P\omega^2 = (3/4)mR^2\omega^2$, so $\omega R = \sqrt{(4gR/3)}$. The lowest point is at a distance 2R from the pivot, so its speed is $2\omega R = \sqrt{(16gR/3)}$.

Answer: (D).

Question 45 (Part A-1, single correct, +3 / -1)

Two rectangular blocks of masses m and 2m are held against a massless compressed spring within a box of mass 3m and length 4L. The box as well as the two blocks are free to move on a horizontal frictionless surface along the x-axis. Just when the blocks leave contact with the spring, each block is at a distance L from the wall of the box in front of it and the centre of the box is at origin x = 0 (see figure). When released, each block moves its own way, collides with the front wall and immediately sticks to it. The position of the centre of the box in the process shifts to Figure: a box of mass 3m and length 4L on a horizontal surface. Inside, a block of mass m on the left and a block of mass 2m on the right are held against a compressed spring at the centre. Each block is at a distance L from the wall in front of it, and the centre of the box is at x = 0.

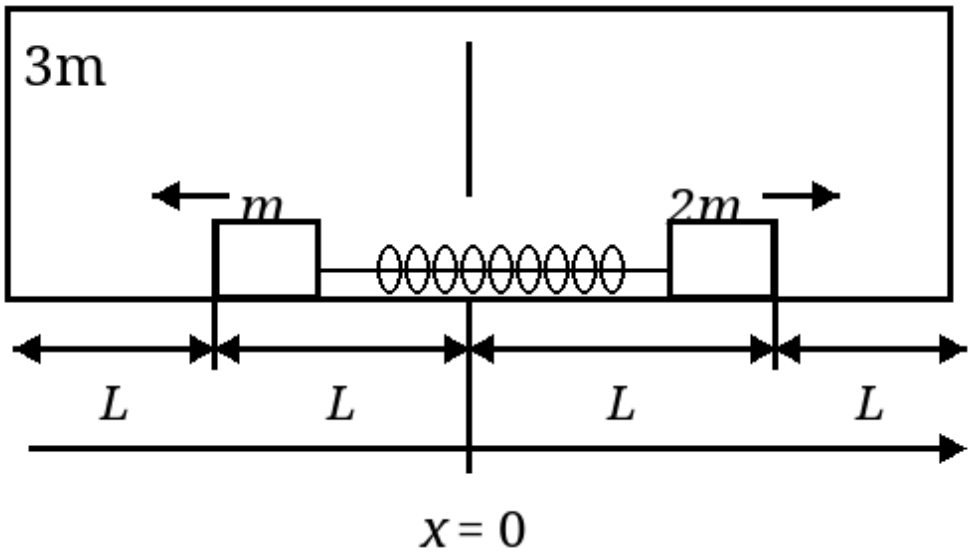


Figure for Question 45 (as printed in the paper).

(a) $x = \frac{L}{2}$ (b) $x = \frac{L}{3}$ (c) $x = \frac{L}{6}$ (d) $x = \frac{L}{6}$

Solution

Correct option: D. **Method:** With no external horizontal force, the centre of mass does not move. Initially, with the box centre at 0, the 2m block at +L and the m block at -L, the centre of mass is at (2mL - mL)/6m = L/6. Finally the blocks stick to the walls at +2L and -2L relative to the box centre. If the box centre shifts by x, then [3mx + 2m(2L + x) + m(-2L + x)]/6m = L/6, giving 6mx = -mL, so x = -L/6.

Answer: (D).

Question 46 (Part A-1, single correct, +3 / -1)

A charge q is uniformly distributed over the surface of a thin circular disk of radius R. The disk rotates about its own axis with an angular velocity ω rad/s, maintaining its axis normal to a uniform magnetic field B. The magnitude of the torque acting on the charged disk is

(a) Zero (b) $\frac{q\omega R^4 B}{4}$ (c) $\frac{q\omega R^2 B}{4}$ (d) $\frac{q\omega R^2 B}{2}$

Solution

Correct option: C. **Method:** The rotating charged disk has magnetic moment $\mu = q\omega R^2/4$ (from $\mu = (q/2m) \times$ angular momentum, or by integrating rings). The disk axis is normal to B, so μ is perpendicular to B and the torque is $\tau = \mu B = q\omega R^2 B/4$.

Answer: (C).

Question 47 (Part A-1, single correct, +3 / -1)

A line source L (perpendicular to the plane of the paper) emitting light of wavelength 5625 Å is placed 1.00 m away directly in front of the slit S_1 in an opaque wall (see figure). A second slit S_2 in the same wall is parallel to slit S_1 and at distance d from it; both the slits are parallel to each other and to the line source L. Intensity of light is measured at the position P, equidistant from each of the slits S_1 and S_2 on the screen, for three different conditions. When S_2 is covered it is I_1, when S_1 is covered it is I_2. When both slits are open it is I. If $I = I_1 + I_2$, the distance d between the two slits is Figure: a line source L on the left, an opaque wall with two slits S_1 (upper) and S_2 (lower) at a distance d apart, and a screen on the right with a point P equidistant from the two slits.

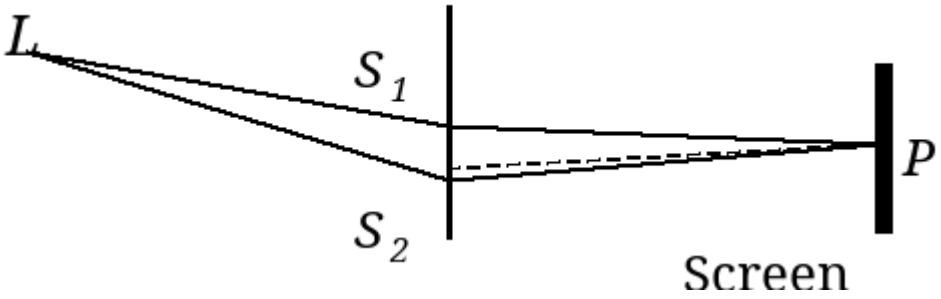


Figure for Question 47 (as printed in the paper).

(a) 0.75 mm (b) 0.53 mm (c) 1.06 mm (d) 1.50 mm

Solution

Correct option: B. **Method:** The total intensity is $I = I_1 + I_2 + 2\sqrt{(I_1 I_2)}\cos\varphi$, so $I = I_1 + I_2$ needs $\cos\varphi = 0$, that is, $\varphi = \pi/2$. The source is directly in front of S_1, so the path from L to S_2 exceeds the path to S_1 by about $d^2/(2D)$, with D = 1.00 m. The phase difference is $\varphi = (2\pi/\lambda)(d^2/2D) = \pi/2$, giving $d^2 = \lambda D/2 = 5625 \times 10^{-10} \times 1/2$, so d = 0.53 mm.

Answer: (B).

Question 48 (Part A-1, single correct, +3 / -1)

$^1H^3$ and $_2He^3$ have almost same atomic mass. $^1H^3$ contains one proton, one electron and 2 neutrons while $_2He^3$ has two protons, two electrons and one neutron. Knowing that the mass of a neutron is slightly greater than that of the proton, the estimation of the nuclear binding energy in the two cases reveals that

(a) the binding energy of the nucleus of $^1H^3$ is equal to that of $_2He^3$ nucleus

(b) the binding energy of the nucleus of $^1H^3$ is less than that of $_2He^3$ nucleus

(c) the binding energy of the nucleus of $^1H^3$ is greater than that of $_2He^3$ nucleus

(d) the data is insufficient to predict any information about the binding energy

Solution

Correct option: C. **Method:** The binding energy is (mass of the separate constituents - mass of the atom)c^2. Both atoms have almost the same mass, but $^1H^3$ is made of heavier constituents (two neutrons instead of one), so its mass defect is larger and its binding energy is greater.

Answer: (C).

PART A-2

Question 49 (Part A-2, multi correct, 6 marks, all-or-nothing)

Charge Q is distributed in non-uniform but spherically symmetric manner with charge density $\rho(r)$ such that $\rho(r) = \rho_0(1 - \frac{r}{R})$ for $r \le R$, and $\rho(r) = 0$ for $r \ge R$. The correct statement(s) is/are

(a) The value of the constant ρ_0 is $\frac{3Q}{2\pi R^3}$

(b) The electric field produced by the given charge has maximum value at $r \frac{2R}{3}$

(c) The maximum electric field produced by this charge distribution is $E \frac{1}{3\pi\varepsilon_0} \frac{Q}{R^2}$

(d) The maximum electric field produced is $E \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}$ at r = R

Solution

Correct options: B, C. **Method:** The total charge is $Q = \int_0^R \rho_0(1 - r/R)4\pi r^2\, dr = \pi\rho_0 R^3/3$, so $\rho_0 = 3Q/(\pi R^3)$, and (a) is wrong. By Gauss's law, $E(r) = (\rho_0/\varepsilon_0)(r/3 - r^2/4R)$; $dE/dr = 0$ gives $r = 2R/3$, so (b) is right. The maximum field is $E_{max} = \rho_0 R/(9\varepsilon_0) = Q/(3\pi\varepsilon_0 R^2)$, so (c) is right. At $r = R$ the field is $Q/(4\pi\varepsilon_0 R^2)$, but that is not the maximum, so (d) is wrong.

Answer: (B), (C).

Question 50 (Part A-2, multi correct, 6 marks, all-or-nothing)

A proton of mass m is released from rest at $x_0 = \frac{\alpha}{\beta}$ and starts moving along x axis under the influence of a conservative force F(x). Its potential energy function is $U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x}$, where α and β are positive constants. Rigorous calculations predict that

(a) the speed of the proton is maximum at $x = 2x_0$

(b) the maximum speed of the proton is $v = \sqrt{\frac{\alpha}{mx_0^2}}$

(c) the force on the proton is zero at $x = 2x_0$

(d) during its motion the proton can move in a region $x_0 \le x \le \infty$

Solution

Correct options: A, C, D. **Method:** $U(x_0) = 0$ and the proton starts from rest, so the total energy is 0. The force is $F = -dU/dx = 2\alpha/x^3 - \beta/x^2$, which is zero at $x = 2\alpha/\beta = 2x_0$, so (c) is right. The potential is minimum there, so the kinetic energy and the speed are maximum there, and (a) is right. The maximum kinetic energy is $-U(2x_0) = \beta^2/(4\alpha)$, giving $v_{max} = \sqrt{(\alpha/(2mx_0^2))}$, so (b) is wrong (it misses the factor $1/\sqrt{2}$). The kinetic energy $-U(x)$ is non-negative for $x \ge x_0$, so (d) is right.

Answer: (A), (C), (D).

Question 51 (Part A-2, multi correct, 6 marks, all-or-nothing)

A long horizontal thin metal wire under tension is attached at one end to a vibrator, which causes transverse oscillations in the wire when the other end of the wire is fastened rigidly. The vibration produces a travelling wave in the wire, which is described by $y = y_0 \sin\pi(\alpha t - \beta x)$, where x is measured in meter along the wire from the vibrator end. Here α and β are known positive constants. The linear density of the wire is μ kg/m. Then

(a) the tension in the wire is $T = \frac{\alpha^2 \mu}{\beta^2}$

(b) the transverse wave propagates in the negative x direction with a speed $v = \frac{\alpha}{\beta}$

(c) the power supplied by the vibrator is $P = \frac{\mu\pi^2\alpha^3 y_0^2}{2\beta}$

(d) if the vibrator vibrates with double the frequency, delivering double power to the same wire under the same tension, the amplitude of the wave will reduce to half of the original value

Solution

Correct options: A, C. **Method:** Comparing with $y = y_0 \sin(\omega t - kx)$ gives $\omega = \pi\alpha$ and $k = \pi\beta$, so the wave speed is $v = \omega/k = \alpha/\beta$ in the +x direction, and (b) is wrong. From $v = \sqrt{(T/\mu)}$, the tension is $T = \mu v^2 = \mu\alpha^2/\beta^2$, so (a) is right. The power is $P = (1/2)\mu\omega^2 y_0^2 v = \mu\pi^2\alpha^3 y_0^2/(2\beta)$, so (c) is right. Doubling the frequency and the power gives $2P = (1/2)\mu(2\omega)^2 A'^2 v$, so $A' = A/\sqrt{2}$, not $A/2$, and (d) is wrong.

Answer: (A), (C).

Question 52 (Part A-2, multi correct, 6 marks, all-or-nothing)

Two non-conducting solid spheres (1) and (2), with uniformly distributed charge throughout their volumes, are held at positions A and B, as shown in the figure. Each sphere has radius R. The centers A and B of the spheres (1) and (2) lie on x axis with the point of contact O at the origin. Given that the total charge on sphere (1) is Q and the electric field E at point P $\frac{R}{2}$, 0 is zero. The correct statement(s) is/are Figure: two solid spheres (1) and (2), each of radius R, touching at the origin O. Sphere (1) has centre A at (-R, 0) and sphere (2) has centre B at (R, 0). The point S is at (-R/2, 0) inside sphere (1) and the point P is at (R/2, 0) inside sphere (2).

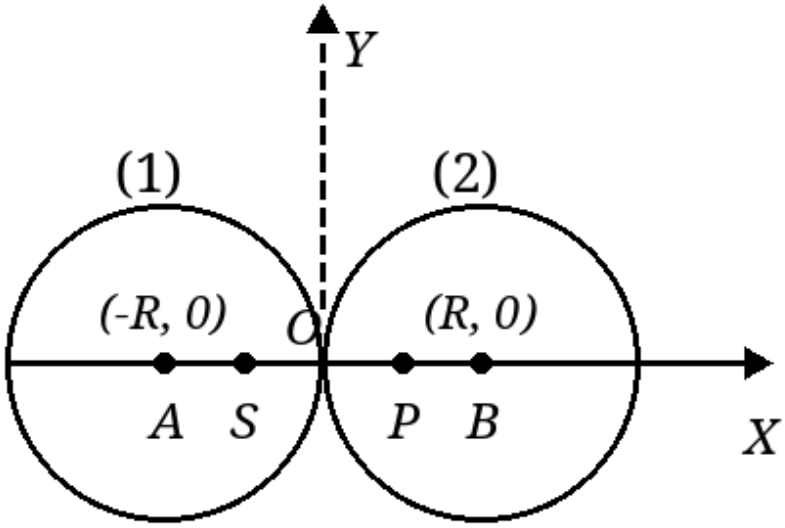


Figure for Question 52 (as printed in the paper).

(a) The charge on sphere (2) is $Q_2 \frac{8Q}{9}$

(b) The ratio of the magnitudes of the electric fields at O and at B is 4 : 9

(c) The electric potential at B is $V_B = \frac{1}{8\pi\varepsilon_0}\frac{Q}{R}$ [taking V = 0 at infinity]

(d) The electric field at the point S $(\frac{R}{2}, 0)$ is in positive x-direction

Solution

Correct options: A, B, D. **Method:** At P, the field from sphere (1) (outside, at distance 3R/2) must balance the field from sphere (2) (inside, at distance R/2): $Q/(4\pi\varepsilon_0(3R/2)^2) = Q_2(R/2)/(4\pi\varepsilon_0 R^3)$, giving $Q_2 = 8Q/9$, so (a) is right. At O, both spheres contribute their surface fields: $|E(O)| = (Q - 8Q/9)/(4\pi\varepsilon_0 R^2) = Q/(36\pi\varepsilon_0 R^2)$. At B, only sphere (1) contributes: $|E(B)| = Q/(4\pi\varepsilon_0(2R)^2) = Q/(16\pi\varepsilon_0 R^2)$. The ratio is 16/36 = 4/9, so (b) is right. The potential at B is $Q/(8\pi\varepsilon_0 R) + (3/2)(8Q/9)/(4\pi\varepsilon_0 R) = 11Q/(24\pi\varepsilon_0 R)$, not $Q/(8\pi\varepsilon_0 R)$, so (c) is wrong. At S, the field from sphere (1) (inside) is $Q/(8\pi\varepsilon_0 R^2)$ in +x and from sphere (2) (outside) is $8Q/(81\pi\varepsilon_0 R^2)$ in -x; the net field points in +x, so (d) is right.

Answer: (A), (B), (D).

Question 53 (Part A-2, multi correct, 6 marks, all-or-nothing)

Two moles of a mixture of a monoatomic gas and a diatomic gas are being used as the working substance in a reversible Carnot cycle as shown in figure. For the mixture of gases, the value of $\gamma_{mixture} = C_P/C_V = \frac{3}{2}$. The cycle operates between the source temperature 600 K and the sink temperature 300 K. The highest and lowest pressures during the cycle are 1600 kPa and 100 kPa. The correct statement(s) is/are Figure: a P-V diagram of the Carnot cycle. A(P_1, V_1) to B(P_2, V_2) is an isotherm at T_1 = 600 K, B to C(P_3, V_3) is an adiabat, C to D(P_4, V_4) is an isotherm at T_2 = 300 K, and D back to A is an adiabat.

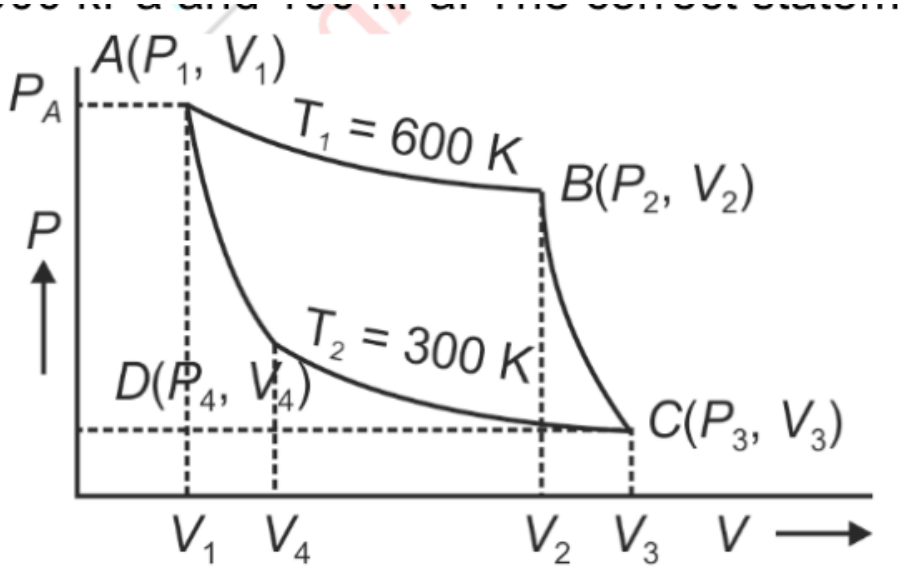


Figure for Question 53 (as printed in the paper).

(a) If the minimum volume in the cycle is V, then the maximum volume is 8V

(b) The working substance consists of one mole of monoatomic and one mole of diatomic gas

(c) Work done per cycle by the engine is 600R ln 2

(d) The minimum volume of working substance is 6.23 litre

Solution

Correct options: A, B, C, D. **Method:** The highest pressure is at A (1600 kPa, 600 K) and the lowest at C (100 kPa, 300 K). For the adiabat B to C, $(T_1/T_2)^{\gamma/(\gamma-1)} = P_B/P_C$, giving $P_B = 800$ kPa; the isotherm A to B then gives $V_B = 2V$. For the isotherm C, PV/T is constant, so $V_C = 8V$, and (a) is right. From $\gamma_{mix} = 3/2$ with 2 moles total, the mixture is 1 mole monoatomic and 1 mole diatomic, so (b) is right. The work per cycle is $W = 2R \times 600 \ln 2 - 2R \times 300 \ln 2 = 600R \ln 2$, so (c) is right. The minimum volume is at A: $V = nRT/P = 2 \times 8.314 \times 600/(1.6 \times 10^6) = 6.23 \times 10^{-3}$ m^3 = 6.23 litre, so (d) is right.

Answer: (A), (B), (C), (D).

Question 54 (Part A-2, multi correct, 6 marks, all-or-nothing)

In the circuit shown, E is the emf of an ideal battery. Capacitor C is initially uncharged. Switch S is closed at time t = 0. The time dependent current through the resistances R_1 and R_2 are I_1 and I_2 respectively. At any moment t, the charging current through the capacitor is $I_C(t)$. Choose correct option(s) Figure: a circuit with an ideal battery E and a switch S in series with a resistor R_1. After R_1, a capacitor C and a resistor R_2 are connected in parallel. The currents are I_1 through R_1, I_C through the capacitor and I_2 through R_2.

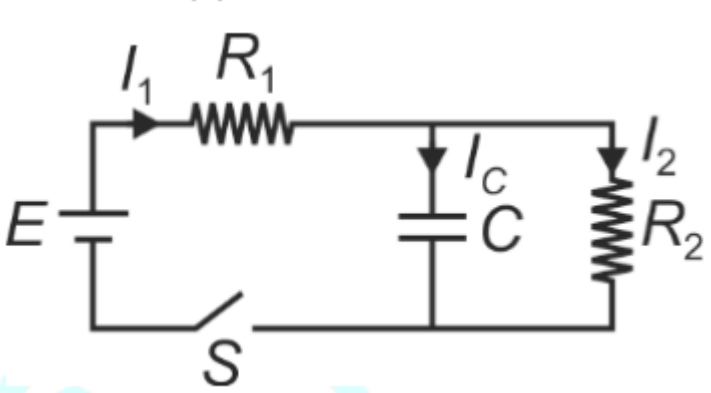


Figure for Question 54 (as printed in the paper).

(a) Current I_C charging the capacitor is given by $I_C = \frac{E}{R_1} e^{-t/CR}$ where $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

(b) Current I_C charging the capacitor is given by $I_C = \frac{E}{R} e^{-t/CR}$ where $R = R_1 + R_2$

(c) Soon after the switch S is closed (at t = 0), the currents are $I_1 \frac{E}{R_1}$ and $I_2 = 0$

(d) Long time after the switch S is closed ($t \to \infty$), charge on the capacitor is q = CE

Solution

Correct options: A, C. **Method:** The resistance seen by the capacitor is the Thevenin resistance R_1 in parallel with R_2, so $1/R = 1/R_1 + 1/R_2$ and the time constant is CR. The charging current is $I_C = (E/R_1)e^{-t/CR}$ (it is E/R_1 at t = 0 and zero at $t = \infty$), so (a) is right and (b) is wrong. At t = 0 the uncharged capacitor short-circuits R_2, so $I_2 = 0$ and $I_1 = E/R_1$, and (c) is right. At $t = \infty$ the capacitor voltage is $ER_2/(R_1 + R_2)$, so $q = CER_2/(R_1 + R_2)$, not CE, and (d) is wrong.

Answer: (A), (C).

Question 55 (Part A-2, multi correct, 6 marks, all-or-nothing)

In the given circuit, the emf E of the battery is 16 V and its internal resistance is r = 1.5 Ω. The correct option(s) is/are Figure: a circuit with a battery E = 16 V of internal resistance 1.5 Ω, in series with a resistor X = 2 Ω, feeding a network between junctions A (top) and C (bottom). Between A and C there are two balanced Wheatstone bridges: a left branch of 6 Ω + 12 Ω, a middle 10 Ω resistor to junction B, and a right network with 1 Ω, 2 Ω, 8 Ω, 4 Ω and 2 Ω resistors joining A, B and C.

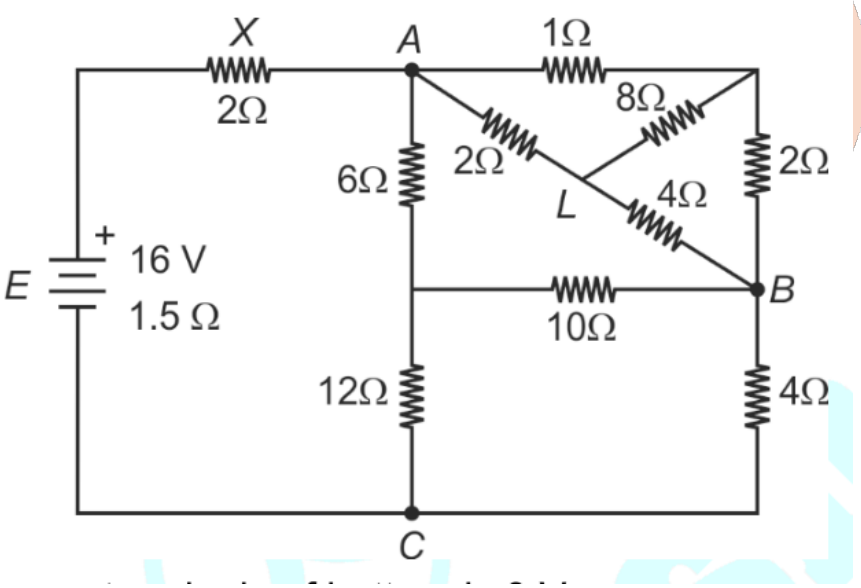


Figure for Question 55 (as printed in the paper).

(a) The potential difference across terminals of battery is 3 V

(b) The power dissipated in the resistance X (= 2 Ω) is 8 W

(c) The power dissipated in 12 Ω resistance is 3 W

(d) The potential difference across junctions A and B is 3 V

Solution

Correct options: B, C, D. **Method:** The network between A and C has two balanced Wheatstone bridges. Between A and B, the 1 Ω + 2 Ω path is in parallel with the 2 Ω + 4 Ω path, so $R_{AB} = (3 \times 6)/9 = 2$ Ω. Between A and C, the 6 Ω + 12 Ω branch is in parallel with the A-B-C path of 2 + 4 = 6 Ω, so $R_{AC} = (18 \times 6)/24 = 4.5$ Ω. The total resistance is 1.5 + 2 + 4.5 = 8 Ω, so the total current is 16/8 = 2 A. The terminal potential difference is 16 -

$1.5 \times 2 = 13$ V, so (a) is wrong. The power in X is $2^2 \times 2 = 8$ W, so (b) is right. The current in the 12 Ω branch is $2 \times 6/24 = 0.5$ A, so its power is $0.25 \times 12 = 3$ W, and (c) is right. The current through the A-B network is 1.5 A, so $V_{AB} = 1.5 \times 2 = 3$ V, and (d) is right.

Answer: (B), (C), (D).

Question 56 (Part A-2, multi correct, 6 marks, all-or-nothing)

A train, moving in a straight line with an initial speed u ms^{-1} at t = 0, is given a constant acceleration 'a' ms^{-2} to attain a maximum speed $V_m = \frac{7u}{3}$. The driver runs the train at that maximum speed for some duration and then decelerates at a rate of 'a' ms^{-2} to attain the same initial speed u ms^{-1} at t = T. The average speed of the train for the entire journey (from time t = 0 to t = T) is 2u. Choose the correct statement(s)

(a) The train runs at the maximum speed for a duration of $T\sqrt{(1-\frac{4u}{aT})}$

(b) The train runs at the maximum speed for a duration of $T\sqrt{(1-\frac{2u}{aT})}$

(c) The train runs at the maximum speed for a duration of $\frac{T}{2}$

(d) The acceleration is $\frac{16u}{3T}$

Solution

Correct options: A, C, D. **Method:** The v-t graph is a trapezium from u up to 7u/3, flat, and back to u. The acceleration time is $t_1 = (V_m - u)/a = 4u/(3a)$. The distance is the area, $uT + (V_m - u)(T - t_1)$, and the average speed is 2u, so $u + (4u/3)(1 - 4u/(3aT)) = 2u$, giving $a = 16u/(3T)$, and (d) is right. Then $t_1 = T/4$, so the time at maximum speed is $T - 2t_1 = T/2$, and (c) is right. With $a = 16u/(3T)$, $T\sqrt{(1 - 4u/(aT))} = T\sqrt{(1 - 3/4)} = T/2$, so (a) is right, while (b) gives $T\sqrt{(5/8)}$, which is wrong.

Answer: (A), (C), (D).

Question 57 (Part A-2, multi correct, 6 marks, all-or-nothing)

A rectangular block of mass M projected up an inclined plane, set at an angle θ with the horizontal, comes to a stop at a certain height and slides back down the incline. The down trip takes exactly twice as long as the up trip. The observations reveal that Figure: a rectangular block of mass M on an inclined plane set at an angle θ to the horizontal.

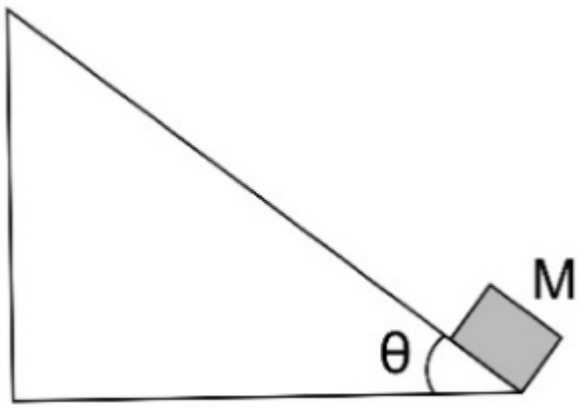


Figure for Question 57 (as printed in the paper).

(a) coefficient of sliding friction between the block and the surface of the incline is 0.6 tan θ

(b) coefficient of sliding friction between the block and the surface of the incline is 0.6 sin θ

(c) about 75% of the original energy of projection has been lost to friction during the entire trip

(d) the acceleration of the block during its sliding down is 0.4 g sin θ

Solution

Correct options: A, C, D. **Method:** Up the plane, $a_1 = g(\sin\theta + \mu\cos\theta)$; down the plane, $a_2 = g(\sin\theta - \mu\cos\theta)$. For the same distance, $t_2 = 2t_1$ gives $a_1 = 4a_2$, so $\sin\theta + \mu\cos\theta = 4(\sin\theta - \mu\cos\theta)$, and $5\mu\cos\theta = 3\sin\theta$, so $\mu = 0.6\tan\theta$. Hence (a) is right and (b) is wrong. Then $a_2 = g(\sin\theta - 0.6\sin\theta) = 0.4\,g\sin\theta$, so (d) is right. The final kinetic energy is $a_2/a_1 = 1/4$ of the initial, so 75% of the energy is lost to friction, and (c) is right.

Answer: (A), (C), (D).

Question 58 (Part A-2, multi correct, 6 marks, all-or-nothing)

A wire loop OABC, with corners at O (0, 0), A (L, 0), B (L, L) and C (0, L) lying in XY plane, carries a current I in anti-clockwise direction as shown. The correct statement(s) is/are Figure: a square wire loop OABC in the XY plane, with O at the origin, A at (L, 0), B at (L, L) and C at (0, L). The current I flows anticlockwise: O to A to B to C to O, as shown by arrows.

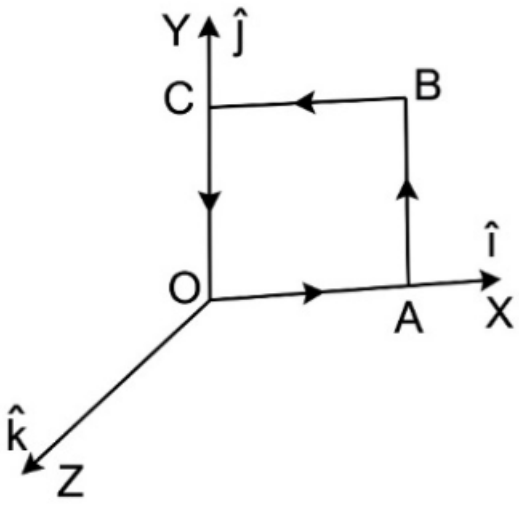


Figure for Question 58 (as printed in the paper).

(a) The magnetic field produced at the centre of the loop is $B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}\,\hat{k}$

(b) The magnetic moment of the current loop is $\mu_0 I L^2 \hat{k}$

(c) If a non-uniform magnetic field $B_2 = \frac{B_0 z}{L}\,\hat{j} + \frac{B_0 y}{L}\,\hat{k}$ is switched on, the magnetic force on the loop is $B_0 I L\,\hat{j}$

(d) The magnetic force on the current carrying loop due to the magnetic field B_2 is zero

Solution

Correct options: A, C. **Method:** Each side of the square produces a field $\mu_0 I\sqrt{2}/(2\pi L)$ at the centre, all in the same direction, so the total is $B_1 = 2\sqrt{2}\mu_0 I/(\pi L)\,\hat{k}$, and (a) is right. The magnetic moment is $IL^2\hat{k}$, with no μ_0, so (b) is wrong. For B_2, the forces on the four sides are $F_{OA} = 0$, $F_{AB} = (IB_0L/2)\hat{i}$, $F_{BC} = IB_0L\hat{j}$ and $F_{CO} = -(IB_0L/2)\hat{i}$, giving a total force of $B_0 IL\hat{j}$, so (c) is right and (d) is wrong.

Answer: (A), (C).

Question 59 (Part A-2, multi correct, 6 marks, all-or-nothing)

A rocket is fired from Earth at 60° to the local vertical with an initial speed $v_0 = \sqrt{(gR)}$ where R is radius of the Earth and g is the acceleration due to gravity at the surface of the Earth. Choose correct option(s) [Assume the Earth to be a uniform solid sphere. Also ignore the rotation of the Earth]

(a) The maximum distance reached by the rocket from the Earth's centre is 2R

(b) The maximum distance reached by the rocket from the Earth's centre is $\frac{3R}{2}$

(c) The minimum velocity of the rocket during the motion is $\sqrt{\frac{gR}{3}}$

(d) The maximum height attained by the rocket above the Earth's surface is $\frac{R}{3}$

Solution

Correct options: B, C. **Method:** The specific energy is $v_0^2/2 - GM/R = gR/2 - gR = -gR/2$, so the semi-major axis is $a = -GM/(2\varepsilon) = R$. The angular momentum is $h = Rv_0 \sin 60°$, giving eccentricity $e = \sqrt{(1 + 2\varepsilon h^2/G^2M^2)} = 1/2$. The maximum distance is $r_{max} = a(1 + e) = 3R/2$, so (b) is right and (a) is wrong. The minimum speed is at apogee: $v_{min} = h/r_{max} = \sqrt{(gR/3)}$, so (c) is right. The maximum height above the surface is $r_{max} - R = R/2$, not $R/3$, so (d) is wrong.

Answer: (B), (C).

Question 60 (Part A-2, multi correct, 6 marks, all-or-nothing)

A uniform magnetic field B = 0.5 T exists in a circular region of radius R = 5 m. A wire loop of radius R = 5 m completely encloses the magnetic field at t = 0. The loop is pulled at uniform speed v = 2 ms^{-1} in the plane of the paper as shown. Figure: a uniform magnetic field B = 0.5 T exists in a circular region (dashed circle, marked with crosses) of radius R = 5 m. A wire loop of the same radius (solid circle) completely encloses the field region at t = 0 and is pulled to the right at a uniform speed v = 2 ms^{-1}, so the two circles move apart.

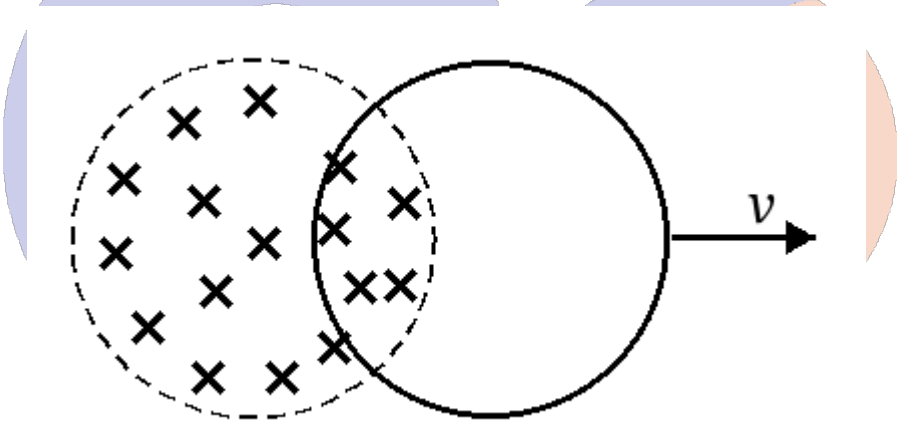


Figure for Question 60 (as printed in the paper).

(a) The emf induced in the loop at time t = 3 s will be 8 V

(b) The maximum emf induced in the loop will be 10 V

(c) The emf induced in the loop will be minimum at time t = 4 s

(d) The emf induced in the loop will be maximum just at start at the time t = 0 s

Solution

Correct options: A, B, D. **Method:** The induced emf is B times the rate of change of the overlap area of the two circles, which is B × v × (chord length). The centres separate by 2t, so the chord length is $2\sqrt{(25 - t^2)}$ and $\varepsilon = 0.5 \times 2 \times 2\sqrt{(25 - t^2)} = 2\sqrt{(25 - t^2)}$ volt. At t = 3 s, ε = 2 × 4 = 8 V, so (a) is right. The maximum is at t = 0, ε = 10 V, so (b) and (d) are right. The emf falls to zero at t = 5 s, not 4 s, so (c) is wrong.

Answer: (A), (B), (D).