

COMPETISHUN

SUBJECT: MATHEMATICS STANDARD (041)

CLASS X (2026 – 27)

Sample Question Paper with Solutions and Marking Scheme**Maximum Marks: 80****Time Allowed: 3 hours****General Instructions**

Read the following instructions very carefully and strictly follow them:

- (i) This Question paper contains 38 questions. All questions are compulsory.
- (ii) This Question paper is divided into five Sections – A, B, C, D and E.
- (iii) In Section A, Question number 1 to 18 are Multiple Choice Questions (MCQs) and Question number 19 and 20 are Assertion - Reason based questions of 1 mark each.
- (iv) In Section B, Question number 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
- (v) In Section C, Question number 26 to 31 are Short Answer (SA)-type questions carrying 3 marks each.
- (vi) In Section D, Question number 32 to 35 are Long Answer (LA)-type questions carrying 5 marks each.
- (vii) In Section E, Question number 36 to 38 are case study-based questions carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C and 2 questions in Section D. An internal choice has been provided in all the sub parts having 2 marks of questions in section E.
- (ix) Draw neat and clean figures wherever required. Take $\pi = 22/7$ wherever required, if not stated.
- (x) Use of calculators is not allowed.

SECTION - A**(20 × 1 = 20)**

This section comprises of 18 multiple choice questions and two questions of assertion and reasoning type of 1 mark each.

Question 1**[1 mark]**

The greatest number which divides both 134 and 188, leaving remainders 4 and 6 respectively, is:

- (A) 13 (B) 26 (C) 39 (D) 65

Answer and Solution**(B) 26**

$$134 - 4 = 130 \text{ and } 188 - 6 = 182$$

$$\text{HCF } (130, 182) = 26$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 2**[1 mark]**

If $f(x) = px^2 + qx + r$, $p \neq 0$ and $p + r = q$, then one of the zeroes of $f(x)$ is:

- (A) q/p (B) r/p (C) $-r/p$ (D) $-q/p$

Answer and Solution**(C) – r/p**Substituting $q = p + r$ in $f(x)$

$$f(x) = px^2 + (p + r)x + r = (x + 1)(px + r)$$

Zeroes are -1 or $-r/p$.**Marking Scheme***1 mark for the correct option (no part marks).***Question 3****[1 mark]**

Tarun correctly solved a pair of linear equations in two variables and found their only point of intersection as $(5, -1)$. One of the lines was $x - y = 6$. Which of the following could have been the other line? I: $3x - 3y = 18$ II: $2x - 3y = 13$ III: $2x - 3y = 16$

- (A) I only (B) II only (C) I and II (D) II and III

Answer and Solution**(B) II only**

$(5, -1)$ is a solution of I: $3x - 3y = 18$ also. But this will make the system of equations as consistent and dependent.

Only II: $2x - 3y = 13$ fulfills the given condition of having only one point of intersection.

Marking Scheme*1 mark for the correct option (no part marks).***Question 4****[1 mark]**

If $(1 - p)$ is a root of the quadratic equation $x^2 + px + 1 - p = 0$, then its roots are:

- (A) 0, 1 (B) $-1, 1$ (C) 0, -1 (D) $-1, 2$

Answer and Solution**(C) 0, -1**

As $(1 - p)$ is a root of the quadratic equation $x^2 + px + 1 - p = 0$,

$$\therefore (1 - p)^2 + p(1 - p) + 1 - p = 0 \Rightarrow p = 1$$

Now the quadratic equation will be:

$$x^2 + x = 0 \Rightarrow x(x + 1) = 0$$

So the roots are 0, -1 .

Marking Scheme*1 mark for the correct option (no part marks).***Question 5****[1 mark]**

The middle term of the A.P.: 10, 7, 4, ..., -62 is:

- (A) -26 (B) -29 (C) -32 (D) -35

Answer and Solution**(A) -26**

$$-62 = 10 + (n - 1)(-3) \Rightarrow n = 25$$

Thus $(25 + 1)/2 = 13$ th term is the middle term.

$$\therefore a_{13} = 10 + 12(-3) = -26$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 6

[1 mark]

The perimeters of two similar triangles are 56 cm and 70 cm respectively. If one side of the first triangle is 14 cm, then the corresponding side of the second triangle (in cm) is:

- (A) 5 (B) 7.5 (C) 10 (D) 17.5

Answer and Solution

(D) 17.5

Ratio of perimeters of similar triangles = ratio of their corresponding sides

$$\therefore 56/70 = 14/(\text{corresponding side})$$

corresponding side = 17.5

Marking Scheme

1 mark for the correct option (no part marks).

Question 7

[1 mark]

The point which lies on the perpendicular bisector of the line segment joining the points A (– 3, – 4) and B (3, 4) is:

- (A) (0, 0) (B) (0, 3) (C) (3, 0) (D) (0, 4)

Answer and Solution

(A) (0, 0)

Mid-point of AB will lie on the perpendicular bisector of the line segment joining the points A (– 3, – 4) and B (3, 4).

$$\therefore \text{Coordinates of the mid-point} = ((-3 + 3)/2, (-4 + 4)/2) = (0, 0)$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 8

[1 mark]

If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = 1/\sqrt{3}$, $0^\circ < A + B < 90^\circ$, $A > B$, then the value of A and B respectively are:

- (A) $60^\circ, 30^\circ$ (B) $60^\circ, 45^\circ$ (C) $45^\circ, 15^\circ$ (D) $60^\circ, 15^\circ$

Answer and Solution

(C) $45^\circ, 15^\circ$

$$\tan(A + B) = \sqrt{3} \Rightarrow (A + B) = 60^\circ \quad \dots (i)$$

$$\tan(A - B) = 1/\sqrt{3} \Rightarrow (A - B) = 30^\circ \quad \dots (ii)$$

Solving equations (i) and (ii), we get $A = 45^\circ$ and $B = 15^\circ$.

Marking Scheme

1 mark for the correct option (no part marks).

Question 9**[1 mark]**

T-shirts marked with numbers 4 to 99 are placed in a box. Gunika is fond of numbers. She randomly takes out a T-shirt from this box. The probability that she gets a T-shirt marked with a number that is either a perfect square or a perfect cube is:

- (A) $1/12$ (B) $1/32$ (C) $11/96$ (D) $5/48$

Answer and Solution**(D) $5/48$**

Total outcomes = 96

Perfect square numbers – 4, 9, 16, 25, 36, 49, 64, 81

Perfect cubes are 8, 27, 64; common number is 64

Number of favourable outcomes = 10

$$P(\text{either a perfect square or a perfect cube}) = 10/96 = 5/48$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 10**[1 mark]**

The diameter of a car wheel is 21 cm. The number of complete revolutions it will make in moving 66 km is:

- (A) 10^4 (B) 10^5 (C) 10^6 (D) 10^7

Answer and Solution**(B) 10^5**

$$\text{Total revolutions} = (66 \times 10^5 \times 7) / (22 \times 21) = 10^5$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 11**[1 mark]**

Two cubes each of volume 64 cm^3 are joined end to end to form a solid. The surface area of the resultant cuboid is:

- (A) 192 cm^2 (B) 160 cm^2 (C) 96 cm^2 (D) 80 cm^2

Answer and Solution**(B) 160 cm^2**

Side of cube = 4 cm

Dimensions of resulting cuboid: length = 8 cm, breadth = 4 cm, height = 4 cm.

$$\text{Total surface area} = 2 (8 \times 4 + 4 \times 4 + 8 \times 4) = 160 \text{ cm}^2$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 12**[1 mark]**

The mean age of a combined group of men and women is 35 years. If the mean ages of the men and women are 38 years and 30 years respectively, then the percentage of women in the group is:

- (A) 15 (B) 25.5 (C) 35 (D) 37.5

Answer and Solution**(D) 37.5**

Let the number of men be x and the number of women be y .

$$(38x + 30y)/(x + y) = 35 \Rightarrow 38x + 30y = 35x + 35y \Rightarrow 3x = 5y$$

$$\therefore x/y = 5/3$$

$$\text{Percentage of women} = 3/8 \times 100 = 37.5$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 13**[1 mark]**

Two dice are rolled simultaneously. The probability of getting number less than 4 on each die is:

- (A) $1/4$ (B) $1/9$ (C) $1/36$ (D) $1/6$

Answer and Solution**(A) $1/4$**

Total outcomes = 36

Favourable outcomes = (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

$$P(E) = 9/36 = 1/4$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 14**[1 mark]**

PM is a median of ΔPQR with vertices P (5, - 6), Q (6, 4) and R (0, 0). The length of PM is:

- (A) $2\sqrt{17}$ units (B) $2\sqrt{15}$ units (C) $\sqrt{101}$ units (D) 10 units

Answer and Solution**(A) $2\sqrt{17}$ units**

$$\text{Coordinates of the midpoint M of QR} = ((6 + 0)/2, (4 + 0)/2) = (3, 2)$$

$$\text{Median PM} = \sqrt{(5 - 3)^2 + (-6 - 2)^2} = \sqrt{68} = 2\sqrt{17} \text{ units}$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 15**[1 mark]**

The arc of a circle is of length 6π cm and the sector it bound has an area of 24π cm². The radius of the circle is:

- (A) 4 cm (B) 8 cm (C) 16 cm (D) 18 cm

Answer and Solution**(B) 8 cm**

$$A = \frac{1}{2} l \times r \Rightarrow 24\pi = \frac{1}{2} \times 6\pi \times r \Rightarrow r = 8 \text{ cm}$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 16**[1 mark]**

The mean and median of the data are 35.5 and 32 respectively. The value of mode for this data is:

- (A) 23 (B) 26 (C) 25 (D) 30

Answer and Solution**(C) 25**

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean} = 3(32) - 2(35.5) = 25$$

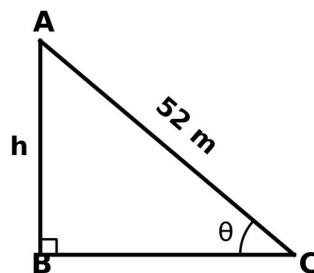
Marking Scheme

1 mark for the correct option (no part marks).

Question 17**[1 mark]**

A flying kite is tied to a point on the ground with the help of a string. The string makes an angle θ with the ground level such that $\tan \theta = 12/5$. If the length of the string is 52 m, then the height of the kite above the ground is:

- (A) 40 m (B) 45.5 m (C) 48 m (D) 50 m

Answer and Solution**(C) 48 m**

Not to scale.

$$\tan \theta = 12/5 \therefore \sin \theta = 12/13$$

$$\Rightarrow h/52 = 12/13 \Rightarrow h = 48 \text{ m}$$

Marking Scheme

1 mark for the correct option (no part marks).

Question 18**[1 mark]**

2 cards of diamonds and 4 cards of spades are missing from a pack of 52 cards. A card is drawn at random from this pack. The probability of getting a card of heart is:

- (A) 13/52 (B) 13/46 (C) 11/52 (D) 11/46

Answer and Solution**(B) 13/46**Total remaining cards = $52 - (2 + 4) = 46$

$$P(\text{a heart card}) = 13/46$$

Marking Scheme

1 mark for the correct option (no part marks).

Assertion-Reason Based Questions

Directions: Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below:

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
- (C) Assertion (A) is true but Reason (R) is false.
- (D) Assertion (A) is false but Reason (R) is true.

Question 19**[1 mark]**

Assertion (A): If three vertices of a parallelogram taken in order are $(-1, -6)$, $(2, -5)$ and $(7, 2)$, then its fourth vertex is $(4, 1)$.

Reason (R): Diagonals of a parallelogram bisect each other.

Answer and Solution

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

The diagonals of a parallelogram bisect each other, so the mid-point of one diagonal is the mid-point of the other.

$$\text{Mid-point of the diagonal joining } (-1, -6) \text{ and } (7, 2) = ((-1 + 7)/2, (-6 + 2)/2) = (3, -2)$$

Taking the fourth vertex as (x, y) , the mid-point of the other diagonal joining $(2, -5)$ and (x, y) must also be $(3, -2)$, which gives $x = 4$ and $y = 1$.

Marking Scheme

1 mark for the correct option (no part marks).

Question 20**[1 mark]**

Assertion (A): If zeroes of the polynomial $(2k - 1)x^2 + 4x - 3$ are reciprocal of each other, then $k = -1$.

Reason (R): If $a = c$, then zeroes of the polynomial $ax^2 + bx + c$, $a \neq 0$ are reciprocal of each other.

Answer and Solution

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

If the zeroes are reciprocals of each other, their product is 1.

$$\therefore c/a = 1, \text{ i.e. } a = c \Rightarrow (-3)/(2k - 1) = 1 \Rightarrow 2k - 1 = -3 \Rightarrow k = -1$$

So Reason (R) is the rule that produces the value stated in Assertion (A).

Marking Scheme

1 mark for the correct option (no part marks).

SECTION – B

(5 × 2 = 10)

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

Question 21

[2 marks]

21 (A)

A hall has a length of 9.75 m, breadth of 6.75 m and height of 5.25 m. What is the length of the longest unmarked ruler that can exactly measure the dimensions of the hall?

OR

21 (B)

There are three bells placed at different swings in a park, which toll at intervals of 5, 6 and 8 minutes, respectively. They all toll together when the park opens for the visitors at 10:00 a.m.. How many more times do they all toll together till the park is closed at 7:00 p.m.?

Solution – 21 (A)

Length = 9.75 m = 975 cm, Breadth = 6.75 m = 675 cm, Height = 5.25 m = 525 cm

Length of longest unmarked ruler = HCF (975, 675, 525)

$$975 = 3 \times 5^2 \times 13, \quad 675 = 3^3 \times 5^2, \quad 525 = 3 \times 5^2 \times 7$$

$$\therefore \text{HCF} (975, 675, 525) = 3 \times 5^2 = 75$$

Hence, the length of the longest unmarked ruler is 75 cm or 0.75 m.

Marking Scheme

Converting the dimensions to cm — $\frac{1}{2}$

Prime factorisation of 975, 675 and 525 — 1

HCF = 75 cm (0.75 m) — $\frac{1}{2}$

Solution – 21 (B)

LCM of 5, 6 and 8 is 120, i.e. all the bells toll together after an interval of 120 minutes or 2 hours.

Thus, all the bells will again toll together at 12:00 noon, 2:00 p.m., 4:00 p.m. and 6:00 p.m., i.e. 4 times.

So, the bells will toll together 4 more times, till the park is closed at 7:00 p.m.

Marking Scheme

LCM of 5, 6 and 8 = 120 minutes (2 hours) — 1

Listing the times and concluding 4 more times — 1

Question 22

[2 marks]

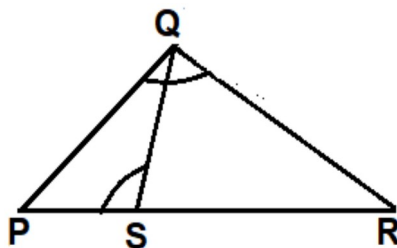
22 (A)

ABCD is a trapezium in which AB is parallel to DC and its diagonals intersect each other at the point O. Show that $AO/BO = CO/DO$.

OR

22 (B)

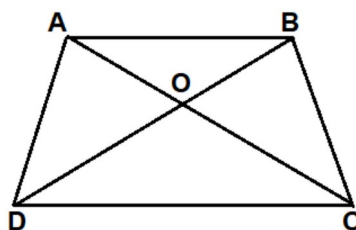
In the given figure, if $\angle PQR = \angle QSP$, $PQ = 6$ cm and $PS = 3$ cm, then find the length of PR .



Not to scale.

For visually impaired candidates: In a right-angled triangle ABC , right-angled at B , a perpendicular BD is drawn to the hypotenuse AC . Prove that the triangle BCD is similar to the triangle ACB .

Solution — 22 (A)



Not to scale.

In $\triangle AOB$ and $\triangle COD$,

$\angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore \triangle AOB \sim \triangle COD$ (AA criterion)

$$\Rightarrow AO/CO = BO/DO \text{ or } AO/BO = CO/DO$$

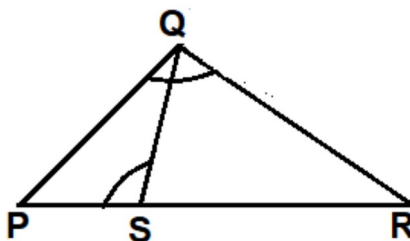
Marking Scheme

Correct figure — 1

Pair of equal angles and the similarity statement — $\frac{1}{2}$

Writing the required proportion — $\frac{1}{2}$

Solution — 22 (B)



Not to scale.

In $\triangle PRQ$ and $\triangle PQS$,

$\angle PQR = \angle QSP$ (Given); $\angle QPR = \angle QPS$ (Common)

$\therefore \triangle PRQ \sim \triangle PQS$ (AA criterion)

$$\Rightarrow PQ/PS = PR/PQ \Rightarrow 6/3 = PR/6$$

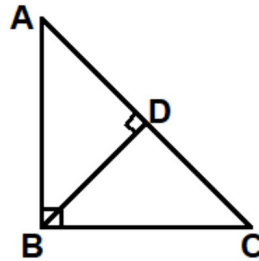
$\therefore \mathbf{PR = 12 \text{ cm}}$

Marking Scheme

Similarity of the two triangles with reasons — 1

Writing the proportion — $\frac{1}{2}$

$PR = 12 \text{ cm}$ — $\frac{1}{2}$

Solution — for visually impaired candidates

Not to scale.

In triangle BCD and triangle ACB,

$\angle BDC = \angle ABC$ (each 90°); $\angle BCD = \angle ACB$ (common)

$\therefore$ Triangle BCD is similar to triangle ACB (AA criterion)

Marking Scheme

First pair of equal angles — $\frac{1}{2}$

Second pair of equal angles — $\frac{1}{2}$

Similarity statement with the AA criterion — 1

Question 23

[2 marks]

If $\tan \theta = 1/\sqrt{5}$, then find the value of $(\operatorname{cosec}^2 \theta - \sec^2 \theta) / (\operatorname{cosec}^2 \theta + \sec^2 \theta)$.

Answer and Solution

$$\tan \theta = 1/\sqrt{5} \Rightarrow \cot \theta = \sqrt{5}$$

$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + (1/\sqrt{5})^2 = 6/5$$

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta = 1 + (\sqrt{5})^2 = 6$$

$$\therefore (\operatorname{cosec}^2 \theta - \sec^2 \theta) / (\operatorname{cosec}^2 \theta + \sec^2 \theta) = (6 - 6/5) / (6 + 6/5) = 2/3$$

Marking Scheme

$\cot \theta = \sqrt{5}$ — $\frac{1}{2}$

Values of $\sec^2 \theta$ and $\operatorname{cosec}^2 \theta$ — $\frac{1}{2}$

Substitution and the value $2/3$ — 1

Question 24

[2 marks]

Find the area of a sector of a circle with diameter 28 cm, if the length of the corresponding arc is 22 cm.

Answer and Solution

$$\text{Radius} = 28/2 = 14 \text{ cm}$$

$$\text{Area of sector} = \frac{1}{2} \times l \times r = \frac{1}{2} \times 22 \times 14$$

$$= 154 \text{ cm}^2$$

Marking Scheme

Radius = 14 cm — $\frac{1}{2}$

Correct formula with substitution — 1

Area = 154 cm^2 — $\frac{1}{2}$

Question 25

[2 marks]

Two dice are thrown simultaneously. Find the probability that the product of the numbers appearing on them is a prime number.

Answer and Solution

Total number of outcomes = 36

Favourable outcomes = 6 : (1, 2), (1, 3), (1, 5), (2, 1), (3, 1), (5, 1)

P (Prime Product) = $6/36 = 1/6$

Marking Scheme

Total number of outcomes = 36 — $\frac{1}{2}$

Listing the 6 favourable outcomes — $\frac{1}{2}$

Probability = $1/6$ — 1

SECTION - C

(6 × 3 = 18)

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

Question 26

[3 marks]

Given that $\sqrt{3}$ is an irrational number, prove that $(2 + 3\sqrt{3})$ is an irrational number.

Answer and Solution

Let us assume that $2 + 3\sqrt{3}$ is a rational number.

Then $2 + 3\sqrt{3} = a/b$, where 'a' and 'b' are integers and $b \neq 0$

$\Rightarrow \sqrt{3} = (a - 2b)/3b$

Since 'a' and 'b' are integers, $(a - 2b)/3b$ is a rational number.

So $\sqrt{3}$ is a rational number, which contradicts the given fact that $\sqrt{3}$ is an irrational number.

Hence, our assumption is wrong. Therefore $2 + 3\sqrt{3}$ is an irrational number.

Marking Scheme

Assumption and writing $2 + 3\sqrt{3} = a/b$ — 1

Obtaining $\sqrt{3} = (a - 2b)/3b$ — 1

Contradiction and conclusion — 1

Question 27

[3 marks]

If α and β are the zeroes of the quadratic polynomial $2x^2 - 8x + 5$, then find the value of $(\alpha + 1/\beta) \times (\beta + 1/\alpha)$.

Answer and Solution

$\alpha + \beta = 4$ and $\alpha\beta = 5/2$

$(\alpha + 1/\beta) \times (\beta + 1/\alpha) = ((\alpha\beta + 1)/\beta) \times ((\alpha\beta + 1)/\alpha) = ((\alpha\beta + 1)^2)/\alpha\beta$

$= ((5/2 + 1)^2)/(5/2) = 49/10$

Marking Scheme

$$\alpha + \beta = 4 \text{ and } \alpha\beta = 5/2 - 1$$

Simplifying the product to $((\alpha\beta + 1)^2)/\alpha\beta - 1/2$

Substituting the values — 1

Final value $49/10 - 1/2$

Question 28**[3 marks]****28 (A)**

Ridhi drew a polygon with n sides. The smallest exterior angle is 8° and each subsequent exterior angle is 4° more than the previous exterior angle. Find the number of sides of the polygon that Ridhi had drawn.

OR**28 (B)**

Find the sum of integers between 1 and 400 that are multiples of 4 as well as of 5.

Solution — 28 (A)

Let the number of sides of the given polygon be ' n '. The exterior angles of the given polygon are in A.P.

Here $a = 8$ and $d = 4$. As the sum of all the exterior angles of a polygon is 360° ,

$$\therefore S_n = 360 \Rightarrow n/2 [2 \times 8 + (n - 1) \times 4] = 360$$

$$\Rightarrow n^2 + 3n - 180 = 0 \Rightarrow (n + 15)(n - 12) = 0$$

$\therefore n = -15$ or 12 . An angle is always positive, $\therefore n = 12$.

So, the number of sides in the given polygon is 12.

Marking Scheme

Identifying the A.P. with $a = 8$, $d = 4$ and $S_n = 360$ — 1

Forming $n^2 + 3n - 180 = 0$ — 1

Solving the equation — $1/2$

Rejecting $n = -15$ and concluding $n = 12$ — $1/2$

Solution — 28 (B)

The required A.P. is 20, 40, 60, ..., 380. Here $a = 20$ and $d = 20$.

$$\text{Let } a_n = 380 \Rightarrow 20 + (n - 1) \times 20 = 380 \Rightarrow n = 19$$

$$\therefore S_{19} = 19/2 [20 + 380] = 3800$$

Marking Scheme

Writing the A.P. 20, 40, ..., 380 with $a = 20$, $d = 20$ — 1

Forming the equation for a_n — $1/2$

$$n = 19 \text{ — } 1/2$$

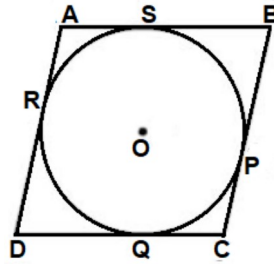
Applying the sum formula — $1/2$

$$\text{Sum} = 3800 \text{ — } 1/2$$

Question 29**[3 marks]**

Prove that the parallelogram circumscribing a circle is a rhombus.

Answer and Solution



Not to scale.

Given, ABCD is a parallelogram circumscribing a circle.

$AS = AR$ --- (i); $BS = BP$ --- (ii); $CQ = CP$ --- (iii); $DQ = DR$ --- (iv)

(Lengths of tangents drawn from an external point to a circle are equal)

Adding (i), (ii), (iii) and (iv), we get

$$AS + BS + CQ + DQ = AR + BP + CP + DR$$

$$\Rightarrow AB + CD = AD + BC \Rightarrow 2AB = 2AD \Rightarrow AB = AD$$

Since adjacent sides of parallelogram ABCD are equal, ABCD is a rhombus.

Marking Scheme

Correct figure with the four equal tangent pairs — 1

Adding the four relations — 1

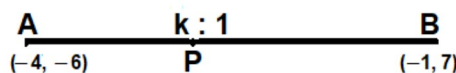
Concluding $AB = AD$ and hence a rhombus — 1

Question 30

[3 marks]

In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Also, find the coordinates of the point of division.

Answer and Solution



Not to scale.

Let the x-axis divide the line segment joining the points A $(-4, -6)$ and B $(-1, 7)$ at P $(x, 0)$ in the ratio $k : 1$.

$$\text{Coordinates of P} = \left(\frac{k(-1) + 1(-4)}{k+1}, \frac{k(7) + 1(-6)}{k+1} \right) = \left(\frac{-k-4}{k+1}, \frac{7k-6}{k+1} \right)$$

Since the point P lies on the x-axis,

$$\therefore \frac{7k-6}{k+1} = 0 \Rightarrow k = 6/7$$

So, point P divides AB in the ratio 6 : 7.

Coordinates of P are $(-34/13, 0)$.

Marking Scheme

Applying the section formula to write the coordinates of P — 1

Equating the y-coordinate to zero — $\frac{1}{2}$

$k = 6/7$, i.e. the ratio 6 : 7 — $\frac{1}{2}$

Coordinates of P — 1

Question 31**[3 marks]****31 (A)**

During a math class, Ms. Isha wrote the expression given below on the board and asked the students to simplify it.

$$(\cos \theta)/(1 - \sin \theta) + (1 - \sin \theta)/(\cos \theta)$$

Jyoti solved it in her note book as follows:

$$\begin{aligned} & (\cos \theta)/(1 - \sin \theta) + (1 - \sin \theta)/(\cos \theta) \\ &= (\cos^2 \theta + (1 - \sin \theta)^2)/((1 - \sin \theta) \times \cos \theta) && \dots \text{ (step 1)} \\ &= (\cos^2 \theta + \cos^2 \theta)/((1 - \sin \theta) \times \cos \theta) && \dots \text{ (step 2)} \\ &= 2\cos^2 \theta/((1 - \sin \theta) \times \cos \theta) && \dots \text{ (step 3)} \\ &= (2\cos \theta)/(1 - \sin \theta) && \dots \text{ (step 4)} \end{aligned}$$

Identify the step in which Jyoti has made error(s), if any. Rectify the same and find the correct answer.

OR**31 (B)**

If $1/(\sin x - \cos x) = (\operatorname{cosec} x)/\sqrt{2}$, then prove that $(1/(\sin x + \cos x))^2 = \sec^2 x/2$.

Solution — 31 (A)

Jyoti made an error in step 2. The correct solution is as under:

$$\begin{aligned} & (\cos \theta)/(1 - \sin \theta) + (1 - \sin \theta)/(\cos \theta) \\ &= (\cos^2 \theta + (1 - \sin \theta)^2)/((1 - \sin \theta) \times \cos \theta) \\ &= (\cos^2 \theta + 1 + \sin^2 \theta - 2 \sin \theta)/((1 - \sin \theta) \times \cos \theta) \\ &= (2 - 2 \sin \theta)/((1 - \sin \theta) \times \cos \theta) \\ &= (2 \times (1 - \sin \theta))/((1 - \sin \theta) \times \cos \theta) = 2/(\cos \theta) = 2 \sec \theta \end{aligned}$$

Marking Scheme

Identifying the error in step 2 — 1
 Correct expansion of $(1 - \sin \theta)^2$ and simplification — 1
 Reaching $(2 - 2 \sin \theta)/((1 - \sin \theta) \times \cos \theta)$ — $\frac{1}{2}$
 Final answer $2 \sec \theta$ — $\frac{1}{2}$

Solution — 31 (B)

$$\text{Given: } 1/(\sin x - \cos x) = (\operatorname{cosec} x)/\sqrt{2} \Rightarrow \sin x - \cos x = \sqrt{2} \sin x$$

Squaring both sides, we get

$$\begin{aligned} 2 \sin x \cos x &= 1 - 2 \sin^2 x \quad \dots (i) \\ \text{Now, LHS} &= (1/(\sin x + \cos x))^2 = 1/(1 + 2 \sin x \cos x) \\ &= 1/(1 + (1 - 2 \sin^2 x)) = 1/(2 \cos^2 x) \\ &= \sec^2 x/2 = \text{RHS} \end{aligned}$$

Marking Scheme

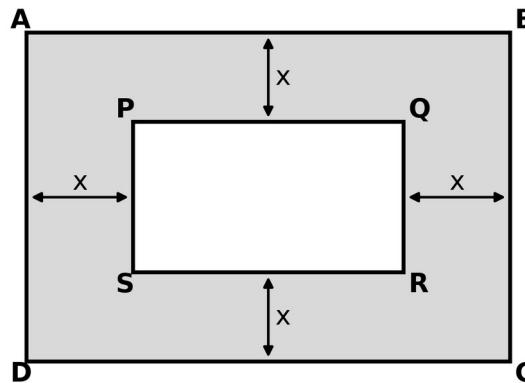
Obtaining $\sin x - \cos x = \sqrt{2} \sin x$ — $\frac{1}{2}$
 Squaring and obtaining relation (i) — 1
 Writing LHS in terms of $2 \sin x \cos x$ — $\frac{1}{2}$
 Substituting from (i) — $\frac{1}{2}$
 Reaching $\sec^2 x/2$ — $\frac{1}{2}$

SECTION - D**(4 × 5 = 20)**

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

Question 32**[5 marks]**

In a rectangular park measuring 50 m × 40 m, the gram panchayat plans to construct a rectangular swimming pool in the middle, surrounded by a uniform-width grass strip throughout the park. Find the dimensions of the swimming pool, if the area of the grassy strip is 1184 m².

Answer and Solution

Not to scale.

ABCD is the rectangular park and PQRS is the swimming pool. Let 'x' be the uniform width of the grass strip. Then, length of the swimming pool, PQ = (50 – 2x) m and breadth of the swimming pool, QR = (40 – 2x) m.

Area of rectangular park – Area of swimming pool = Area of grass strip.

$$50 \times 40 - (50 - 2x)(40 - 2x) = 1184$$

$$\Rightarrow x^2 - 45x + 296 = 0 \Rightarrow (x - 8)(x - 37) = 0 \Rightarrow x = 8, 37$$

Rejecting $x = 37$, as the length and breadth cannot be negative. $\therefore x = 8$

Thus, length of the swimming pool = 50 – 16 = 34 m and breadth = 40 – 16 = 24 m.

Marking Scheme

Correct figure — ½

Expressing the dimensions of the pool in terms of x — 1

Forming the equation from the area of the grass strip — 1

Reducing to $x^2 - 45x + 296 = 0$ — ½

Solving the quadratic equation — 1

Rejecting $x = 37$ — ½

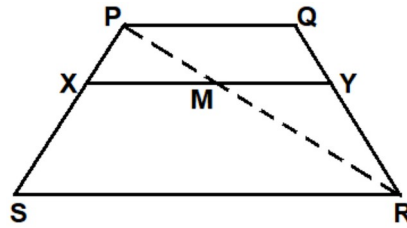
Dimensions 34 m × 24 m — ½

Question 33**[5 marks]**

Prove that a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, divides the other two sides in the same ratio.

Using the above theorem solve the following:

PQRS is a trapezium with $PQ \parallel SR$. X and Y are points on non-parallel sides PS and QR respectively such that $XY \parallel PQ$. Show that $PX/XS = QY/YR$.

Answer and Solution**Basic Proportionality Theorem: correct figure, given, to prove, construction and proof.****Application:***Not to scale.*

Join PR, intersecting XY at M.

$$\text{In } \triangle PSR, XM \parallel SR \therefore PX/XS = PM/MR \text{ --- (i)}$$

$$\text{In } \triangle PQR, MY \parallel PQ \therefore QY/YR = PM/MR \text{ --- (ii)}$$

$$\text{From (i) and (ii), } PX/XS = QY/YR$$

Marking Scheme

Correct figure, given, to prove and construction — 1

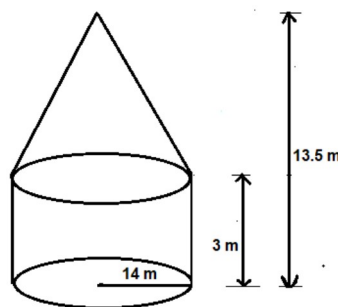
Correct proof of the theorem — 2

Construction: joining PR to meet XY at M — $\frac{1}{2}$ Applying the theorem in $\triangle PSR$ — $\frac{1}{2}$ Applying the theorem in $\triangle PQR$ — $\frac{1}{2}$ Combining (i) and (ii) — $\frac{1}{2}$ **Question 34****[5 marks]****34 (A)**

A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, then find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 250 per m^2 .

OR**34 (B)**

A solid wooden toy is in the form of a hemisphere surmounted by a cone of the same radius. The radius of the hemisphere is 3.5 cm and the total wood used in making the toy is $166 \frac{5}{6} \text{ cm}^3$. Find the height of the conical part. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹ 15 per cm^2 .

Solution — 34 (A)*Not to scale.*

Height of conical part = $13.5 - 3 = 10.5$ m

Slant height (l) = $\sqrt{(14)^2 + (10.5)^2} = 17.5$ m

CSA of conical part = $\pi r l = 22/7 \times 14 \times 17.5 = 770$ m²

CSA of cylindrical part = $2\pi r h = 2 \times 22/7 \times 14 \times 3 = 264$ m²

Total area of canvas = CSA of conical part + CSA of cylindrical part + wastage

= $770 + 264 + 26 = 1060$ m²

Cost of canvas @ ₹ 250 per m² = $250 \times 1060 = ₹ 2,65,000$

Marking Scheme

Height of the conical part — $\frac{1}{2}$

Slant height $l = 17.5$ m — 1

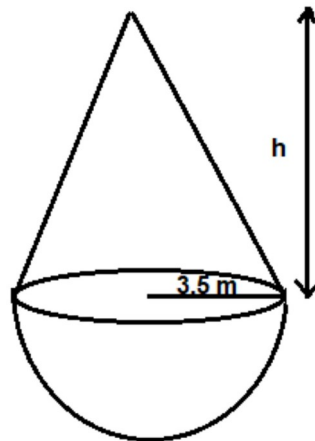
CSA of the conical part — 1

CSA of the cylindrical part — 1

Total canvas = 1060 m² — $\frac{1}{2}$

Cost = ₹ 2,65,000 — 1

Solution — 34 (B)



Not to scale.

Let 'h' cm be the height of the conical part.

Volume of wood in the toy = Volume of conical part + Volume of hemispherical part

$$166\frac{5}{6} = \frac{1}{3} \times \pi \times r^2 \times h + \frac{2}{3} \times \pi \times r^3$$

$$\Rightarrow 1001/6 = \frac{1}{3} \times 22/7 \times (3.5)^2 \times h + \frac{2}{3} \times 22/7 \times (3.5)^3$$

$\Rightarrow h = 6$ cm

$$\text{CSA of hemispherical part} = 2 \times \pi \times r^2 = 2 \times 22/7 \times (3.5)^2 = 77 \text{ cm}^2$$

Cost of painting the hemispherical part @ ₹ 15 per cm² = $15 \times 77 = ₹ 1155$

Marking Scheme

Forming the volume equation and substituting — 2

$h = 6$ cm — 1

CSA of the hemispherical part = 77 cm² — 1

Cost = ₹ 1155 — 1

Question 35

[5 marks]

35 (A)

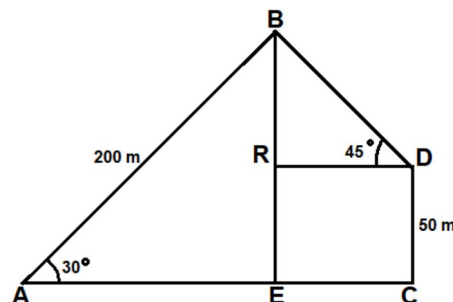
Manish is standing on level ground and observes a kite flying 200 m away from him at an angle of elevation of 30° . Mahesh, standing on the roof of a 50 m high building on the opposite side of the kite, observes the same kite at an angle of elevation of 45° . Find the distance between the kite and Mahesh.

OR

35 (B)

The angles of depression of two ships from the top of a lighthouse and on the same side of it are found to be 45° and 30° . If the ships are 200 m apart and one ship is exactly behind the other, then find the height of the lighthouse.

Solution — 35 (A)



Not to scale.

Let Manish be standing at point 'A' and Mahesh at point 'D'. Distance of the kite from Manish = 200 m.

In rt. angled $\triangle AEB$, $BE/200 = \sin 30^\circ = \frac{1}{2} \Rightarrow BE = 100$ m

$BR = BE - RE = BE - DC = 100 - 50 = 50$ m

In rt. angled $\triangle BRD$, $50/BD = \sin 45^\circ = 1/\sqrt{2} \Rightarrow BD = 50\sqrt{2}$ m

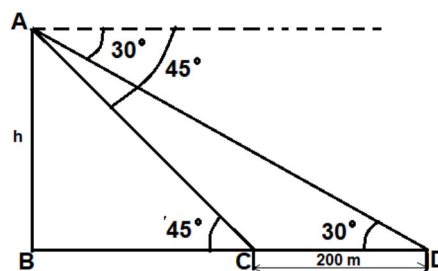
$\therefore$ Distance of the kite from Mahesh is $50\sqrt{2}$ m.

Marking Scheme

Correct figure — 1

Using $\sin 30^\circ$ in $\triangle AEB$ — 1 $BE = 100$ m — $\frac{1}{2}$ $BR = 50$ m — 1Using $\sin 45^\circ$ in $\triangle BRD$ — 1 $BD = 50\sqrt{2}$ m — $\frac{1}{2}$

Solution — 35 (B)



Not to scale.

Let the height of the lighthouse be 'h' m.

In rt. angled $\triangle ABC$, $h/BC = \tan 45^\circ = 1 \Rightarrow BC = h$ --- (i)

In rt. angled ΔABD , $h/(h + 200) = \tan 30^\circ = 1/\sqrt{3}$

$$\Rightarrow h = 200/(\sqrt{3} - 1) \times (\sqrt{3} + 1)/(\sqrt{3} + 1) \Rightarrow h = 100(\sqrt{3} + 1)$$

$\therefore$ **Height of the lighthouse is $100(\sqrt{3} + 1)$ m.**

Marking Scheme

Correct figure — 1

Using $\tan 45^\circ$ to get $BC = h$ — 1

Setting up the equation with $\tan 30^\circ$ — $\frac{1}{2}$

Simplifying by rationalising — 1

$h = 100(\sqrt{3} + 1)$ — $\frac{1}{2}$

Stating the height — 1

SECTION - E

(3 × 4 = 12)

This section comprises of 3 case-study-based questions of 4 marks each with sub parts. Each case study question has three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

Question 36

[4 marks]

A student entrepreneur started a company that manufactures sanitizers in two sizes – small and large. The cost of a small bottle of sanitizer is ₹10 and that of a large bottle is ₹15. In June, the company sold 1000 bottles and recorded a total sale of ₹12,750. Seeing the increased demand, the company decided to increase the price of both sanitizer bottles by ₹2 each. In the next month, the company sold 2500 bottles and recorded a total sale of ₹34,250.

Based on the above information, answer the following questions:

- (i) Form a linear equation in two variables representing the sale for June. [1]
- (ii) Form a linear equation in two variables representing the sale for July. [1]
- (iii) (A) How many sanitizer bottles of each type were actually sold in June? [2]

OR

- (iii) (B) How many sanitizer bottles of each type were sold in July? [2]

Solution

Let the number of small bottles and large bottles be 'x' and 'y' respectively.

(i) $10x + 15y = 12750$ or $2x + 3y = 2550$

(ii) $12x + 17y = 34250$

(iii) (A) According to the question,

$$x + y = 1000 \text{ --- (i); } 10x + 15y = 12750 \text{ --- (ii)}$$

Solving (i) and (ii), we get $x = 450$ and $y = 550$.

Number of small and large bottles sold are 450 and 550 respectively.

OR

(iii) (B) According to the question,

$$x + y = 2500 \text{ --- (i); } 12x + 17y = 34250 \text{ --- (ii)}$$

Solving (i) and (ii), we get $x = 1650$ and $y = 850$.

Number of small and large bottles sold are 1650 and 850 respectively.

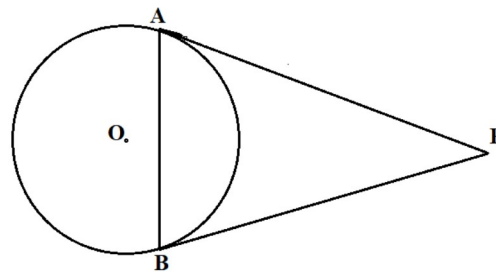
Marking Scheme

- (i) Correct equation — 1
 (ii) Correct equation — 1
 (iii) (A) Forming the pair of equations — 1; solving to get $x = 450$, $y = 550$ — 1
 (iii) (B) Forming the pair of equations — 1; solving to get $x = 1650$, $y = 850$ — 1

Question 37

[4 marks]

In a technology park in Hyderabad, a company installed a circular meditation garden for its employees as shown in the figure.



Not to scale.

Arjun, a designer, stood at a point P outside the garden and drew two tangents PA and PB to the circular boundary. The radius of the garden is 9 m, and the distance of point P from the centre O is 18 m. For preparing safety guidelines, Arjun needed the lengths of the tangents. During a demonstration to interns, he asked them to compute the angle formed between the two tangents using the properties of circle.

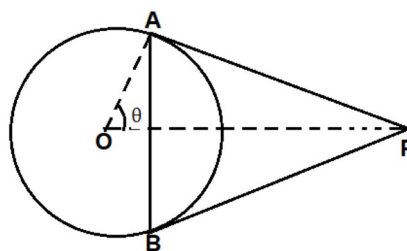
Based on the above information, answer the following questions:

- (i) What is the measure of $\angle OAP$? [1]
 (ii) Calculate the length of the tangent PA. [1]
 (iii) (A) Calculate the measure of $\angle AOB$. [2]

OR

- (iii) (B) In the above design, if PA and PB were inclined at 60° , then what would be their length? [2]

Solution



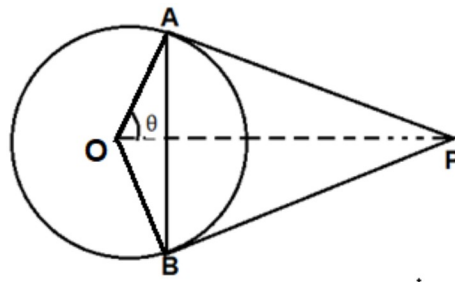
Not to scale.

(i) $\angle OAP = 90^\circ$ (The radius is perpendicular to the tangent at the point of contact)

(ii) In right angled ΔOAP ,

$$OP^2 = OA^2 + AP^2 \Rightarrow (18)^2 = (9)^2 + AP^2 \Rightarrow AP = \sqrt{243} = 9\sqrt{3} \text{ m}$$

(iii) (A)



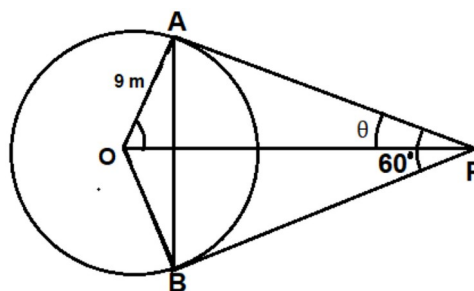
Not to scale.

In right angled ΔOAP , $\cos \theta = \frac{9}{18} = \frac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ$

$\therefore \angle AOB = 2\theta = 120^\circ$

OR

(iii) (B)



Not to scale.

If the tangents are inclined at 60° , then $\angle OPA = \frac{1}{2} \times 60^\circ = 30^\circ$.

In right angled ΔOPA , $\frac{9}{AP} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AP = 9\sqrt{3} \text{ m}$

Marking Scheme

- (i) $\angle OAP = 90^\circ$ — 1
- (ii) Applying Pythagoras theorem — $\frac{1}{2}$; $AP = 9\sqrt{3} \text{ m}$ — $\frac{1}{2}$
- (iii) (A) Finding $\theta = 60^\circ$ — 1; $\angle AOB = 2\theta$ — $\frac{1}{2}$; $\angle AOB = 120^\circ$ — $\frac{1}{2}$
- (iii) (B) $\angle OPA = 30^\circ$ — $\frac{1}{2}$; using $\tan 30^\circ$ — 1; $AP = 9\sqrt{3} \text{ m}$ — $\frac{1}{2}$

Question 38

[4 marks]

A school conducted a weekly test for Class X students, before the commencement of the pre-board examination and recorded their scores (out of 50). To analyse performance patterns, the academic coordinator grouped the marks into intervals. The grouped frequency distribution is as below:

Marks Obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	3	6	12	15	14

The academic coordinator computed the central tendencies to judge overall learning level, the most common performance range and to understand consistency across the batch for planning the remedial sessions.

Based on the above information, answer the following questions:

- (i) Identify the class with the most common performance range. [1]
- (ii) Find the class interval containing the median. [1]
- (iii) (A) What is the average performance of the students? [2]

OR

(iii) (B) Find the mode of the data.

[2]

Solution

(i) 30 – 40

(ii)

Marks	Number of students (f)	cf
0 – 10	3	3
10 – 20	6	9
20 – 30	12	21
30 – 40	15	36
40 – 50	14	50

Since $N/2 = 25$, $\therefore$ Median class is 30 – 40.

(iii) (A)

C.I.	f_i	x_i	$f_i x_i$
0 – 10	3	5	15
10 – 20	6	15	90
20 – 30	12	25	300
30 – 40	15	35	525
40 – 50	14	45	630
Total	50		1560

Average performance, i.e. mean = $1560/50 = 31.2$

OR

(iii) (B) Here, the modal class is 30 – 40, $f_0 = 12$, $f_1 = 15$, $f_2 = 14$.

Mode = $30 + ((15 - 12)/(2 \times 15 - 12 - 14)) \times 10 = 37.5$

Marking Scheme

(i) Most common performance range — 1

(ii) Correct cumulative frequency table and median class — 1

(iii) (A) Correct table — 1; mean = 31.2 — 1

(iii) (B) Identifying the modal class with f_0, f_1, f_2 — $\frac{1}{2}$; substituting in the mode formula — 1; mode = 37.5 — $\frac{1}{2}$